EXTERNAL CONSTRAINTS ON INFLATION THEORIES

MATTEO BRAGLIA · CERN

CosmoForward 2026 · Tenerife · February 10

Funded by
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ChatGPT 5.2

What can I help with?

Are there any non-cosmological constraints on inflation?

Summary: Main Non-Cosmological Constraints

Source	What It Restricts
Particle physics	New fields, couplings
Quantum gravity	Allowed potentials
Naturalness	Fine-tuning
Reheating	Matter creation
Higgs stability	Field fluctuations
EFT consistency	Mathematical health

All of these constrain inflation **before you even look at the sky**.

GRAVITATIONAL WAVE CONSTRAINTS ON INFLATION THEORIES

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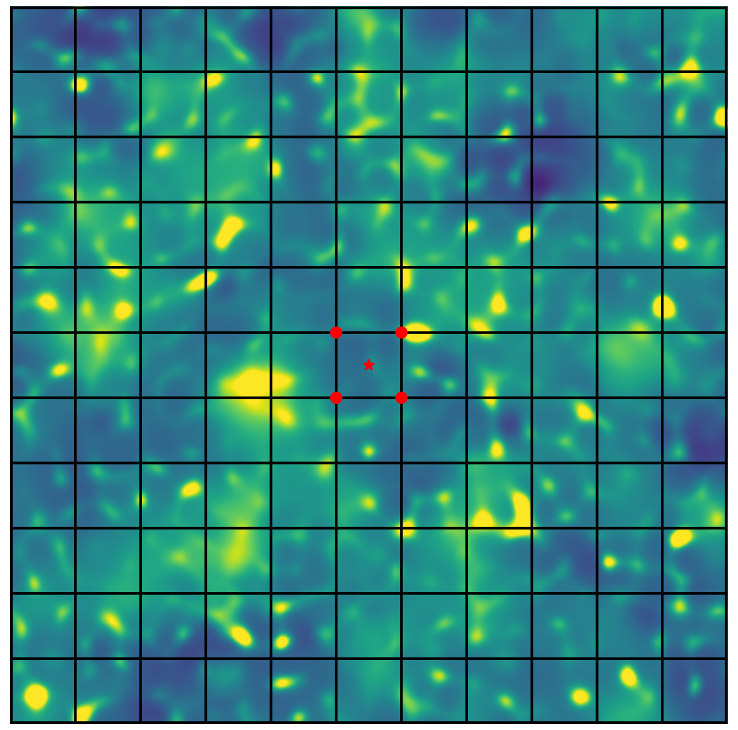
Outline of this talk

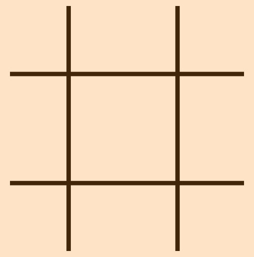
- What we know about inflation, and how we constrain it.
- Current and future constraints from small scales.
- A case study: scalar-induced GWs.

WHAT WE DO KNOW

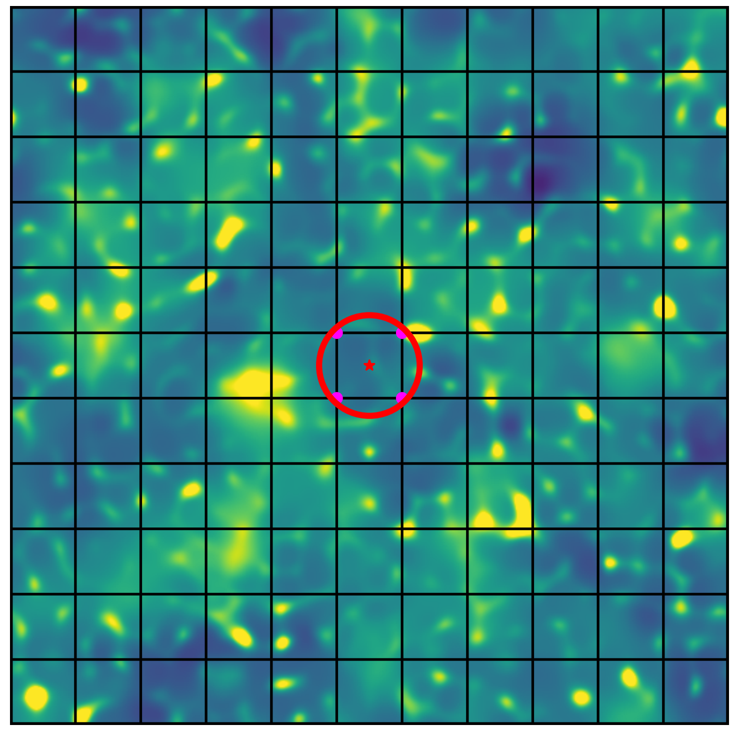
WHAT WE KNOW ABOUT INFLATION



Suppose the Universe starts out highly inhomogeneous



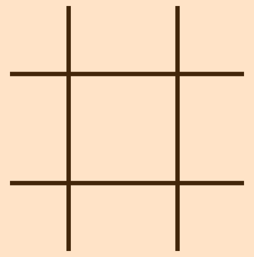
WHAT WE KNOW ABOUT INFLATION



Time t

Suppose the Universe starts out highly inhomogeneous

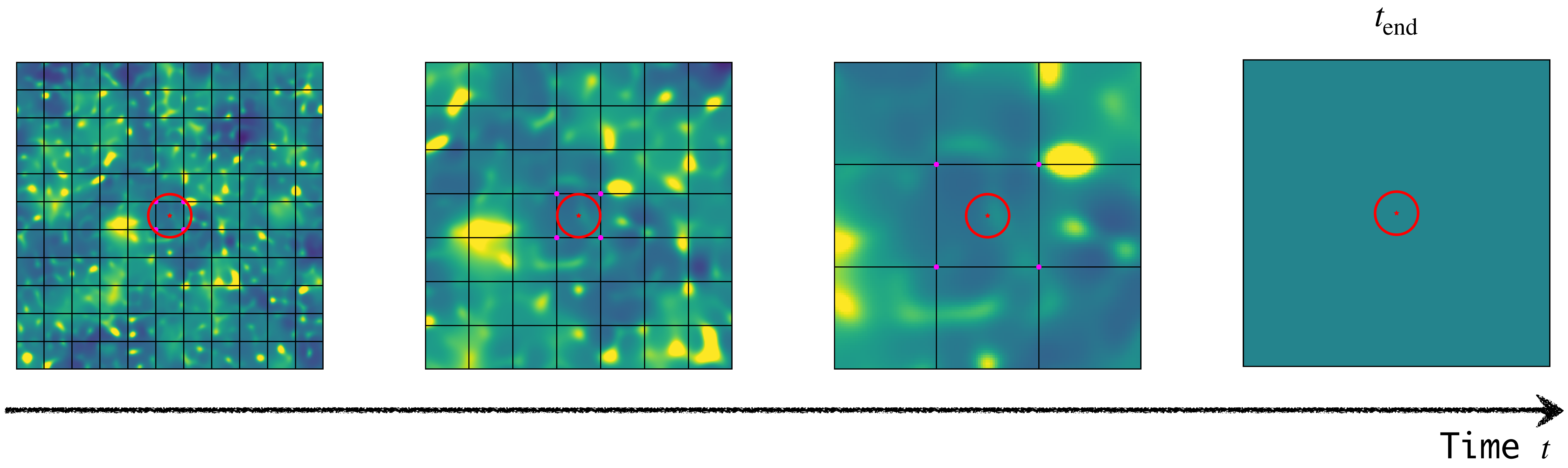
Inflation is a stage of exponential acceleration $a(t) \sim e^{Ht}$ during which $H \sim \mathrm{const}$



: comoving grid.

Hubble radius $H^{-1}(t) \equiv a/\dot{a}$
controls causality
at time t.
If $L_{\mathrm{phys}}(t) < H^{-1}(t)$ particles
can communicate at time t.

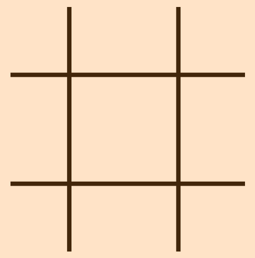
WHAT WE KNOW ABOUT INFLATION



Suppose the Universe starts out highly inhomogeneous

Inflation is a stage of exponential acceleration $a(t) \sim e^{Ht}$ during which $H \sim \mathrm{const}$

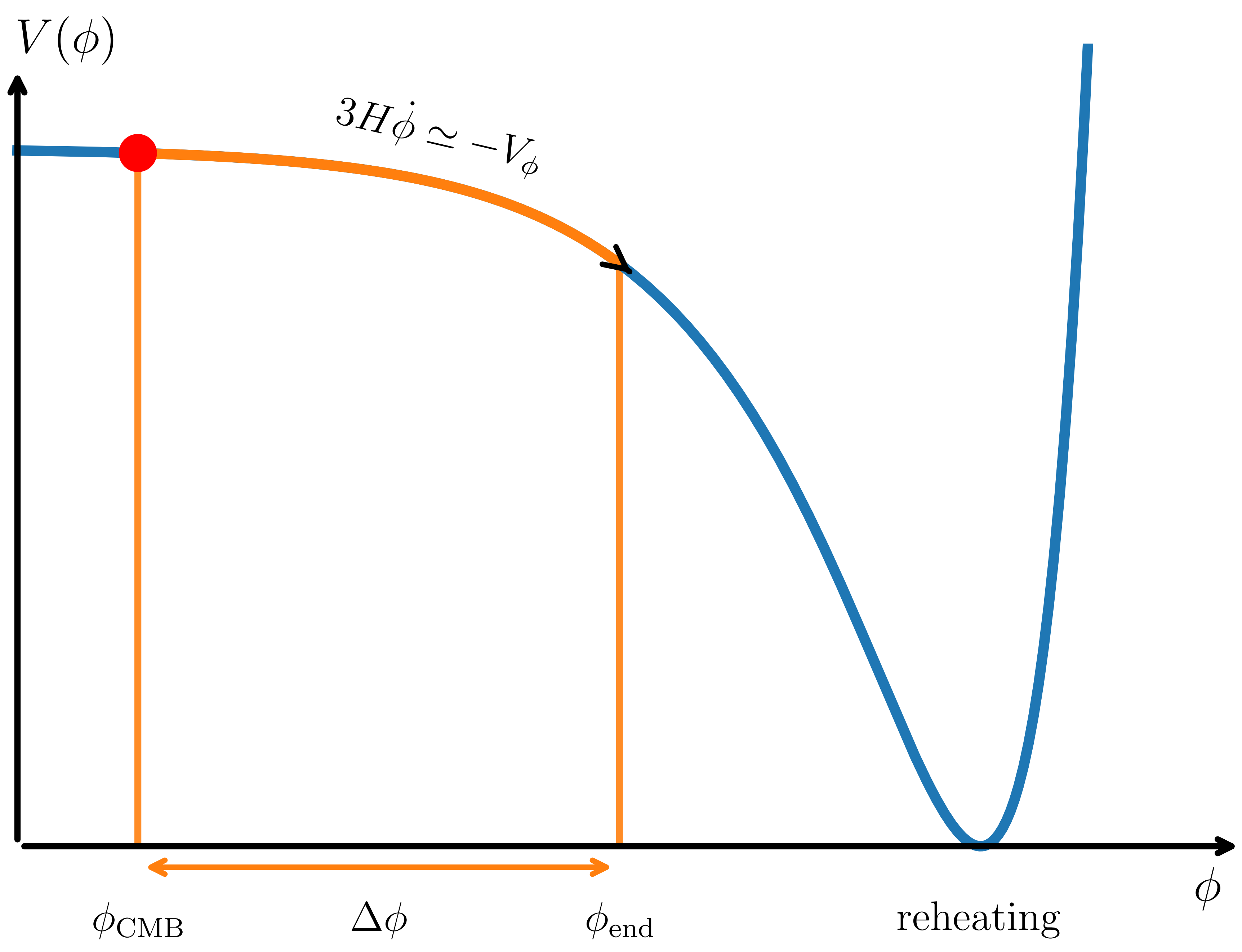
Inhomogeneities are washed out, we dynamically get a highly homogeneous Universe


: comoving grid.


Hubble radius $H^{-1}(t) \equiv a/\dot{a}$
controls causality
at time t.
If $L_{\mathrm{phys}}(t) < H^{-1}(t)$ particles
can communicate at time t.

t_{end} : end of inflation.
It is also the onset
of the Hot Big Bang
evolution for
instantaneous
reheating

WHAT WE KNOW ABOUT INFLATION



Simplest microphysical realization of inflation: $S = \int d^4x \sqrt{-g} \left[-\frac{(\partial\phi)^2}{2} - V(\phi) \right]$

The Universe inflates if $\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{\dot{\phi}^2}{2H^2} \ll 1 \leftrightarrow \ddot{a} > 0$

Inflation lasts long enough if $\epsilon_i \equiv \frac{d \ln \epsilon_{i-1}}{H\, dt} \ll 1$

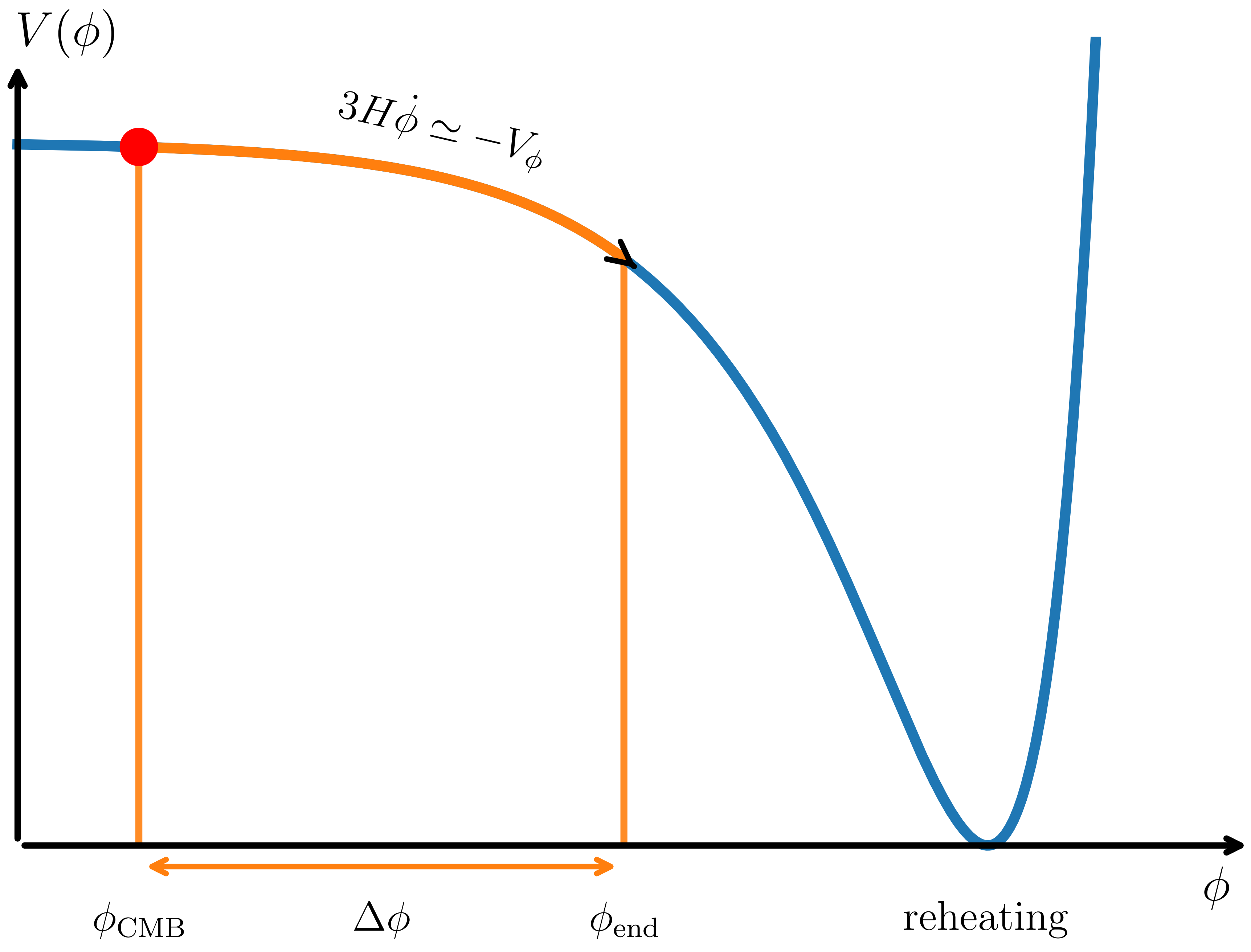
Inflation was proposed by Guth, Linde & Starobinsky in the '70s, and is now a textbook subject. See e.g. Baumann 2022, Dodelson & Schmidt 2020

ϕ : inflaton field. Depends only on time to respect FLRW.

Definition of 'Long' enough: $\sim 10^{-47}s$ of inflation is sufficient to solve the horizon problem, which is equivalent to $N_{\rm tot} = \ln(a_i/a_{\rm end}) \sim 60$ e-folds.

Under the SR conditions, the solution of the KG equation $\phi = \phi(N)$ can be solved analytically once specified $V(\phi)$.

WHAT WE KNOW ABOUT INFLATION



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SLOW-ROLL CONDITIONS

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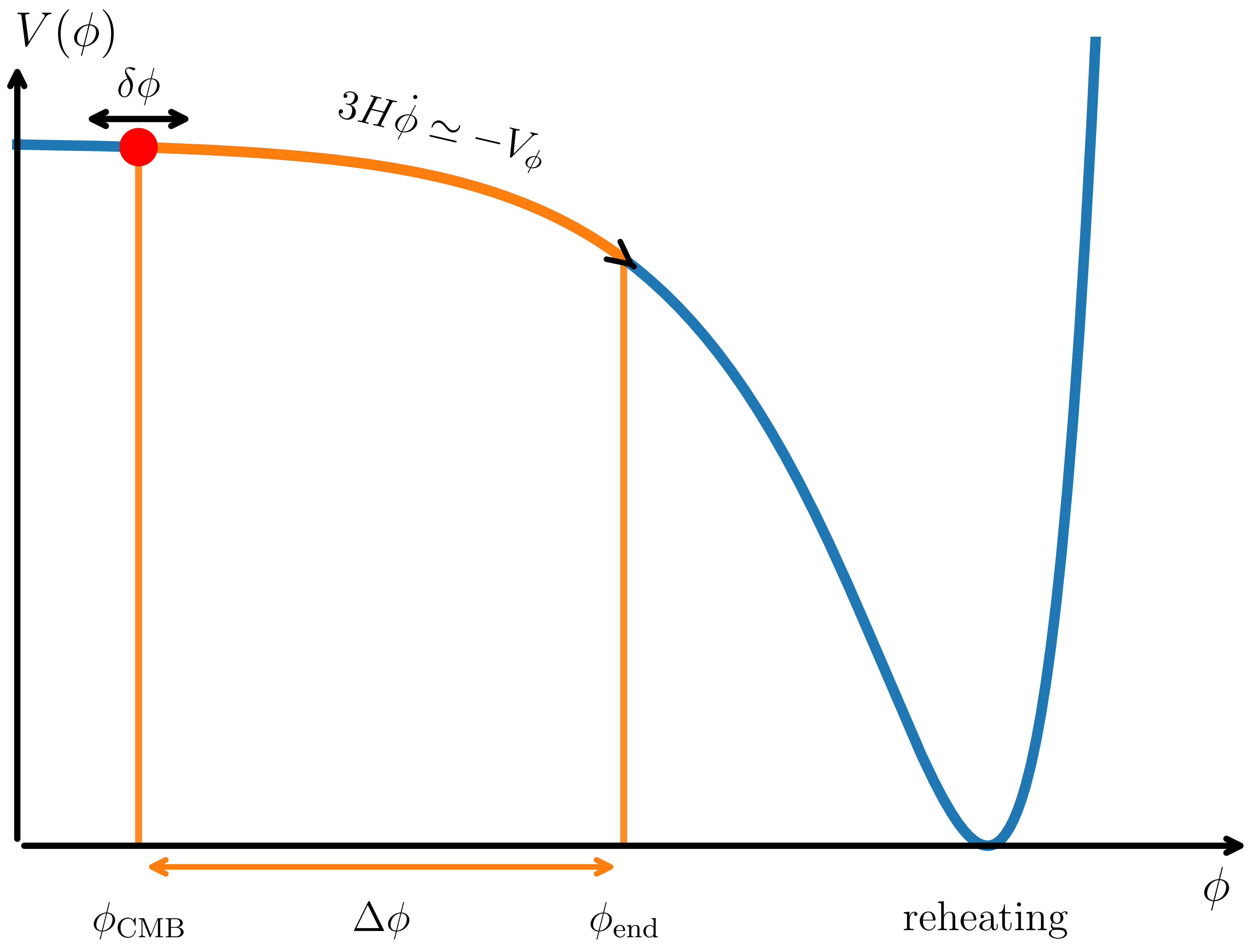
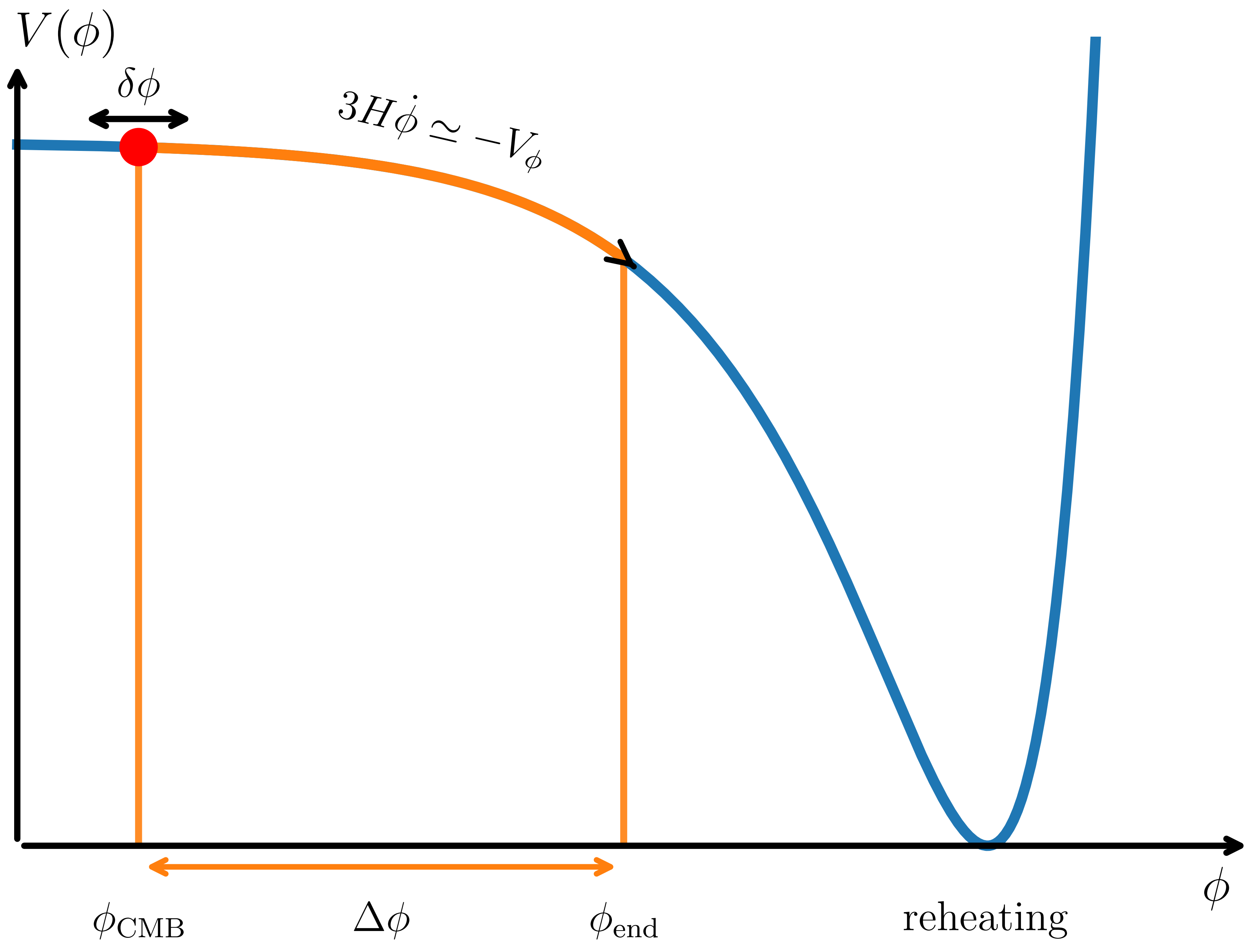
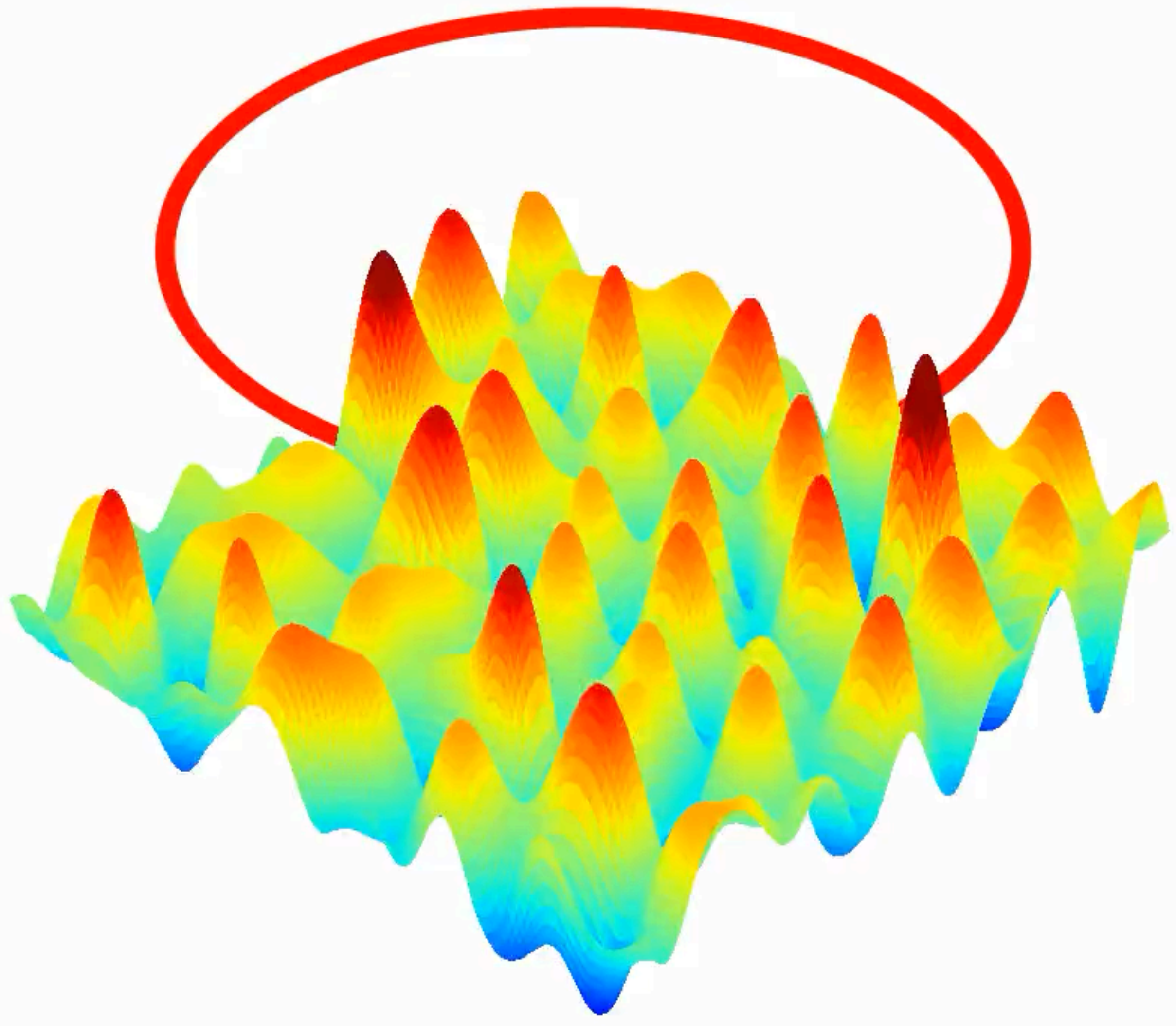


Scalar and tensor fluctuations are excited $ds^2 = -dt^2 + a^2(t)e^{\zeta}(e^{\gamma})_{ij}dx_i dx_j$

The inflaton develops small quantum fluctuations on top of the classical background $\phi(\vec{x},t) = \bar{\phi}(t) + \delta\phi(\vec{x},t)$.

WHAT WE KNOW ABOUT INFLATION





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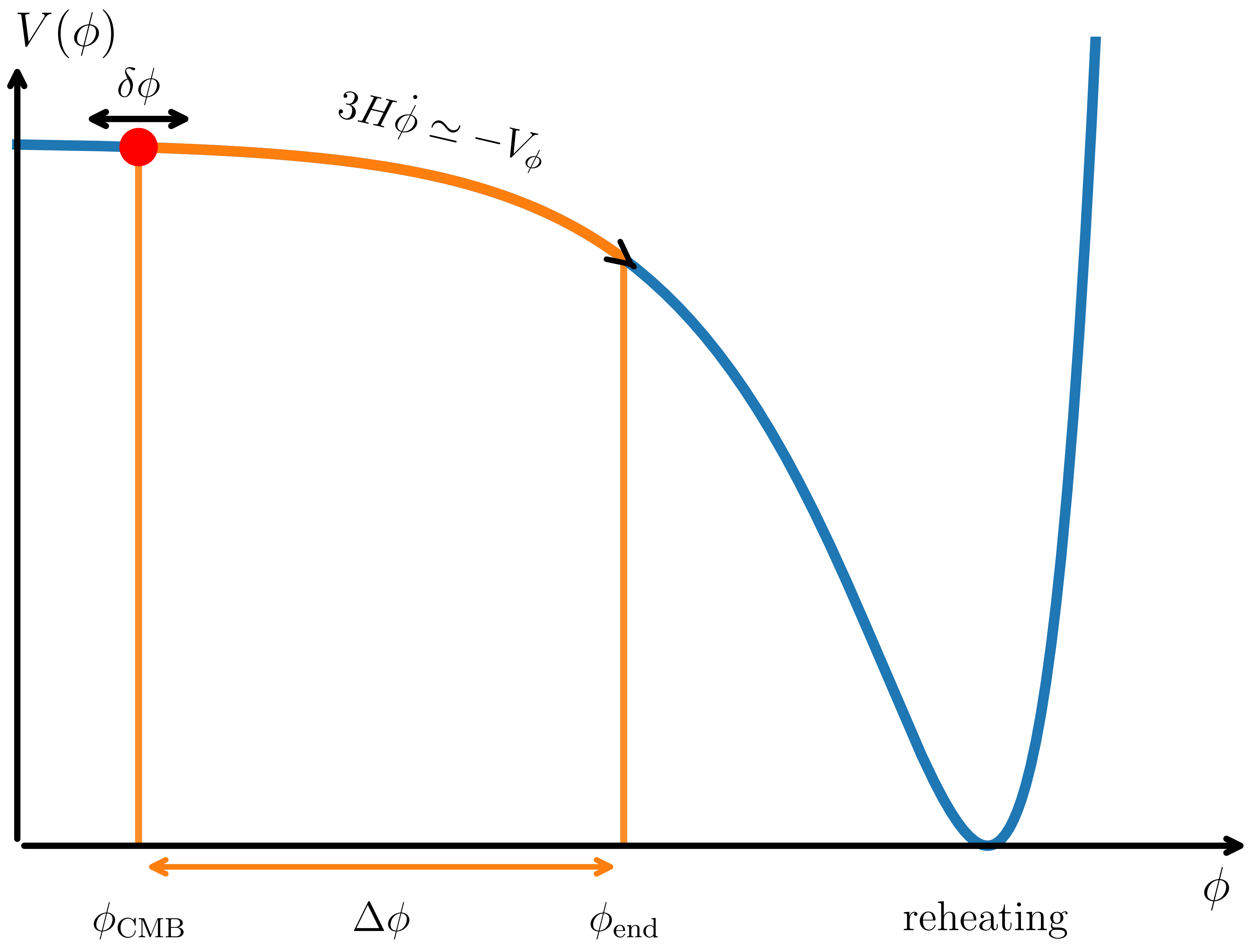
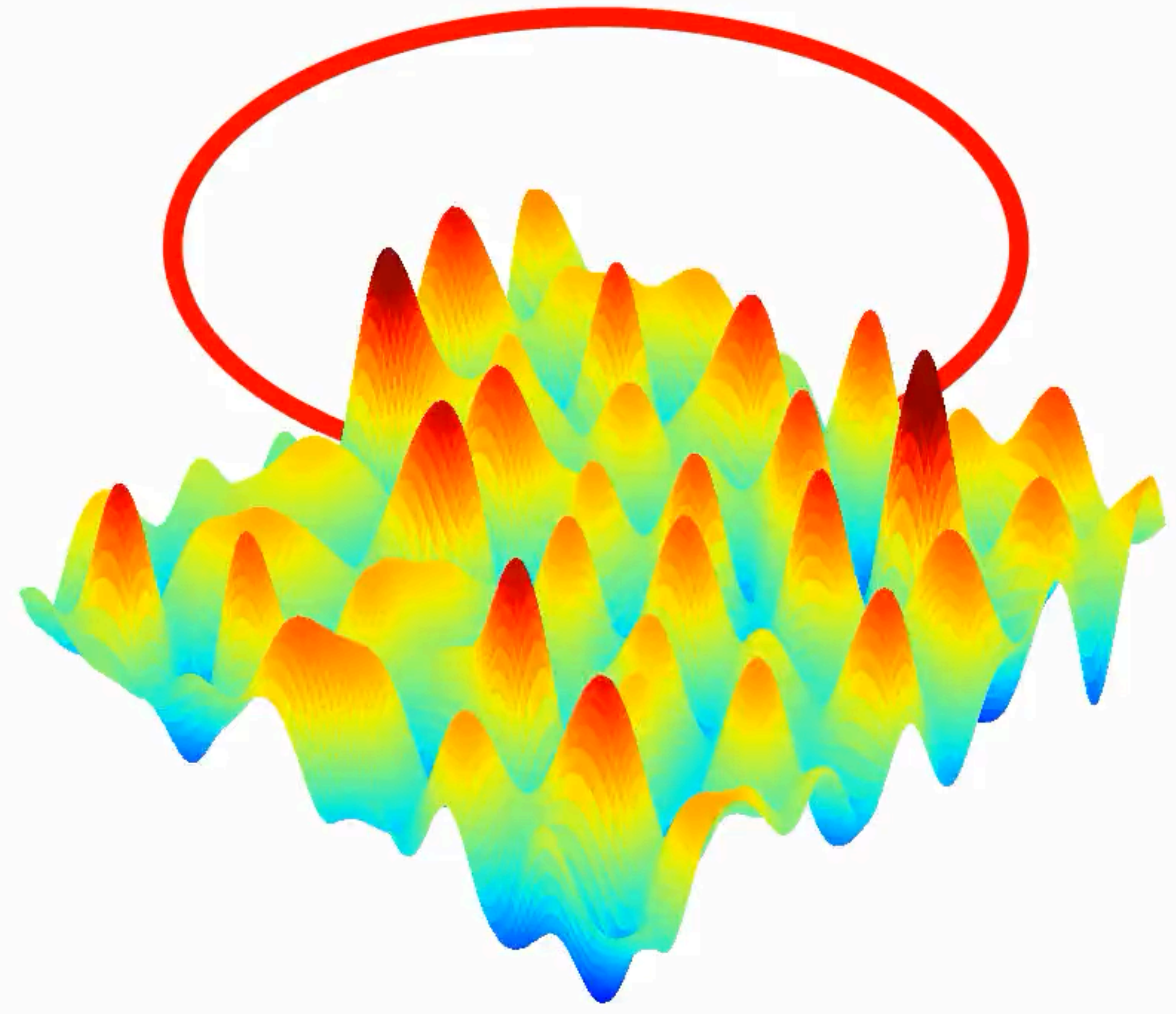
The curvature perturbation is related to the inflaton by $\zeta(\vec{x},t) = H\delta\phi(\vec{x},t)/\dot{\bar{\phi}}$. Like γ, it freezes on super-Hubble scales. The plot shows the evolution of a single Fourier mode.

Scalar and tensor fluctuations are excited $ds^2 = -dt^2 + a^2(t)e^{\zeta}(e^{\gamma})_{ij}dx_i dx_j$

After inflation, these fluctuations source the seeds for CMB and LSS through GR.

WHAT WE KNOW ABOUT INFLATION





Scalar and tensor fluctuations are excited $ds^2 = -dt^2 + a^2(t)e^{\zeta}(e^{\gamma})_{ij}dx_i dx_j$

After inflation, these fluctuations source the seeds for CMB and LSS through GR.

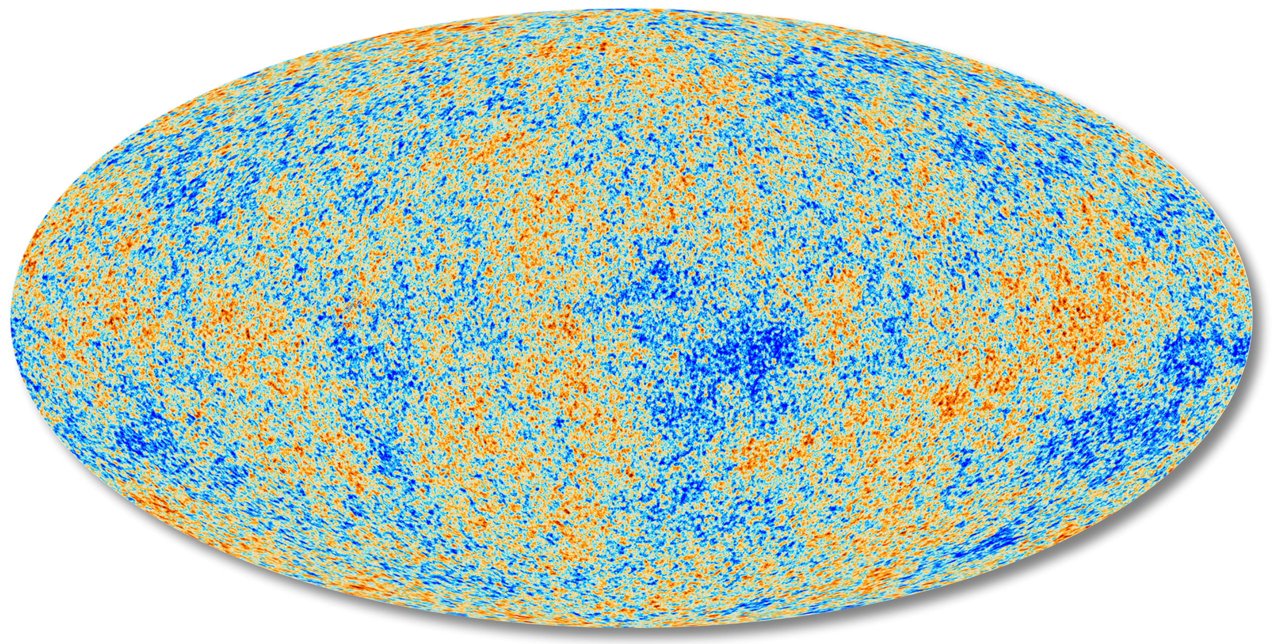
The goal of inflationary theorists is to accurately determine their statistics.

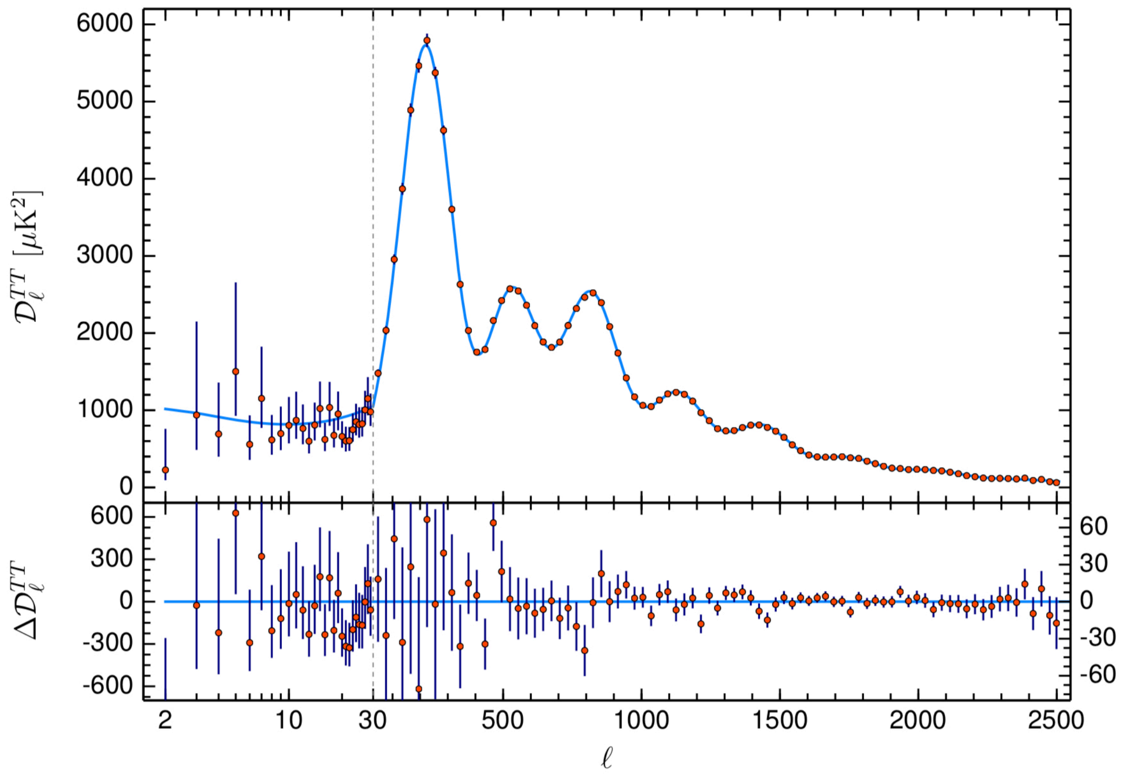
The inflaton develops small quantum fluctuations on top of the classical background $\phi(\vec{x},t) = \bar{\phi}(t) + \delta\phi(\vec{x},t)$.

The curvature perturbation is related to the inflaton by $\zeta(\vec{x},t) = H\delta\phi(\vec{x},t)/\dot{\bar{\phi}}$. Like γ, it freezes on super-Hubble scales. The plot shows the evolution of a single Fourier mode.

Correlation functions of quantum fluctuations can be computed using ‘standard’ QFT in curved space times.

WHAT WE KNOW ABOUT INFLATION



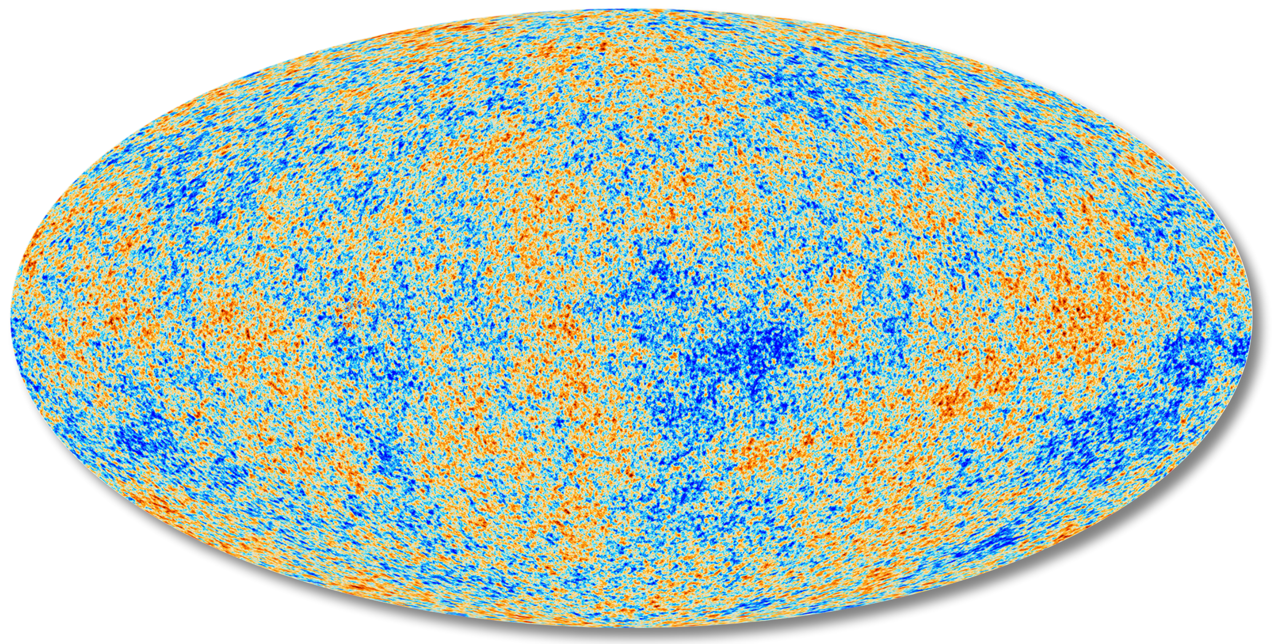


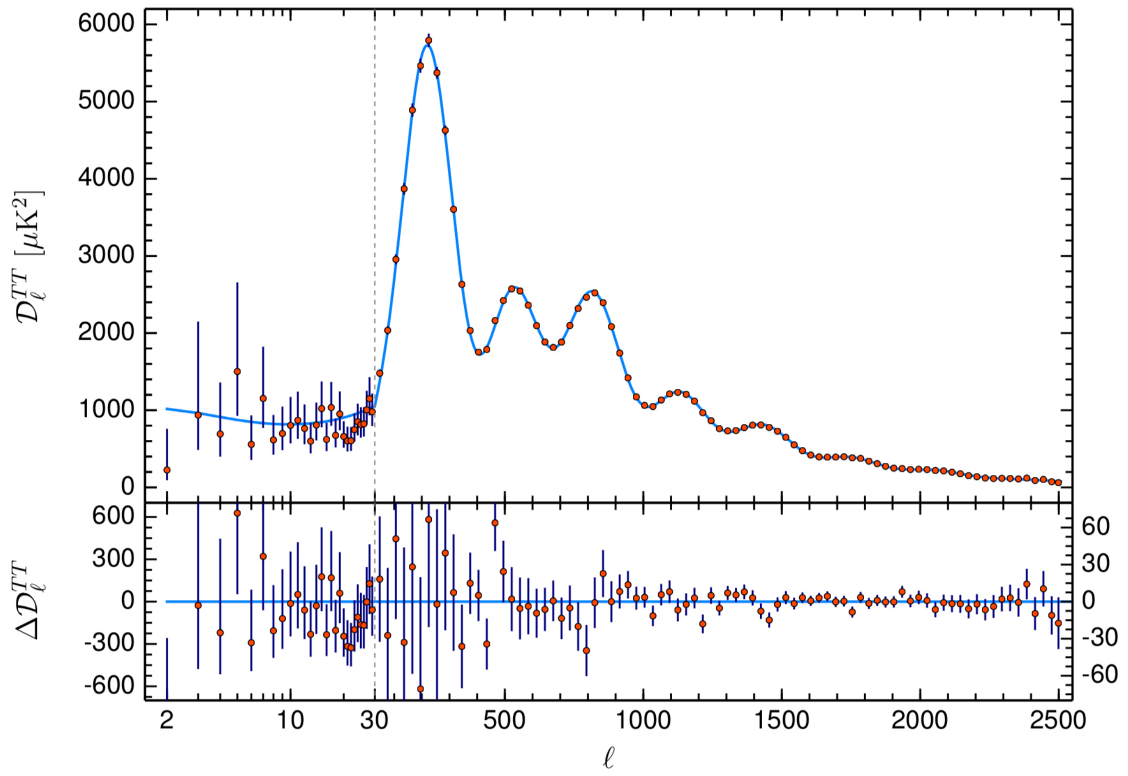
Planck 2018 data

Given the data, one can compare the theory prediction $C_\ell \propto \sum_k T_\ell(k)\mathscr{P}(k)$

CMB is linear, LSS requires non-linear Physics.

WHAT WE KNOW ABOUT INFLATION





Planck 2018 data

Given the data, one can compare the theory prediction $C_\ell \propto \sum_k T_\ell(k)\mathcal{P}(k)$

SR predictions:

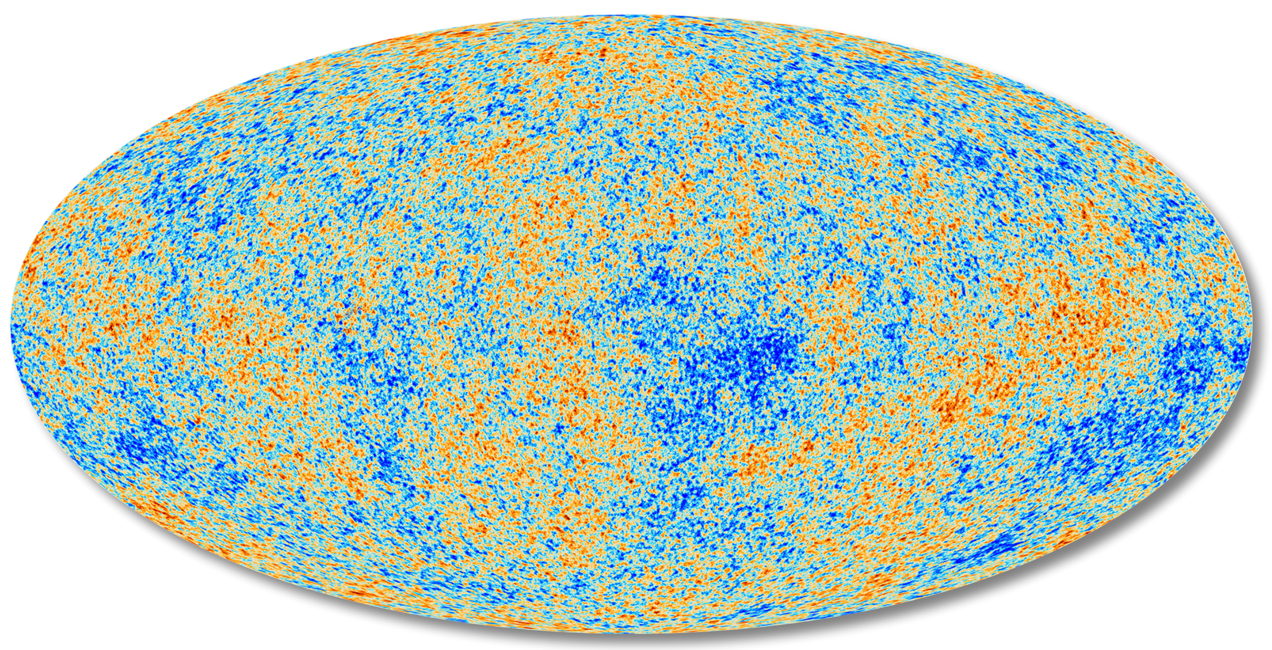
$$\epsilon \ll 1, \quad |\eta| \equiv |\frac{\dot{\epsilon}}{H\epsilon}| \ll 1, \quad |\eta_2| \equiv |\frac{\dot{\eta}}{H\eta}| \ll 1,$$

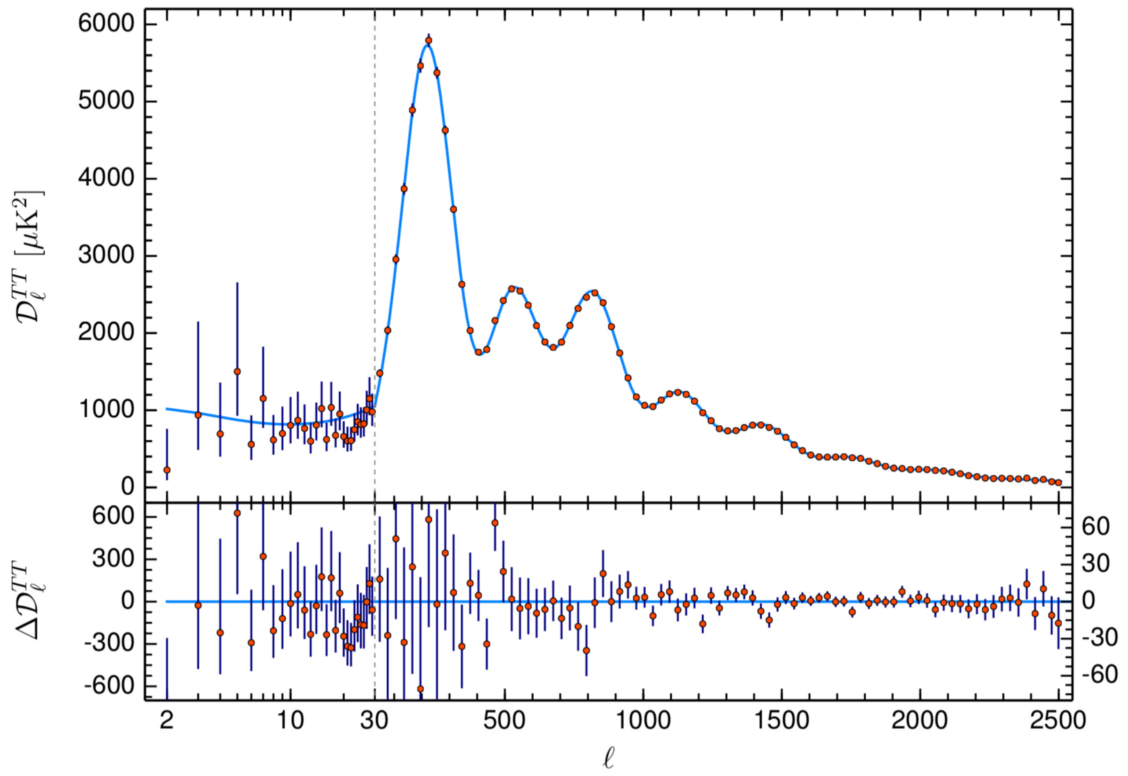
$$\mathcal{P}_\zeta = A_s \left(\frac{k}{k_*}\right)^{n_s - 1}, \quad A_s = \frac{H_*^2}{8\pi^2\epsilon_*}, \quad n_s - 1 = -2\epsilon_* - \eta_*$$

CMB is linear, LSS requires non-linear Physics.

A generic prediction of inflation is the production of GW. The search of this smoking gun is the focus of many of the talks in this conference.

WHAT WE KNOW ABOUT INFLATION





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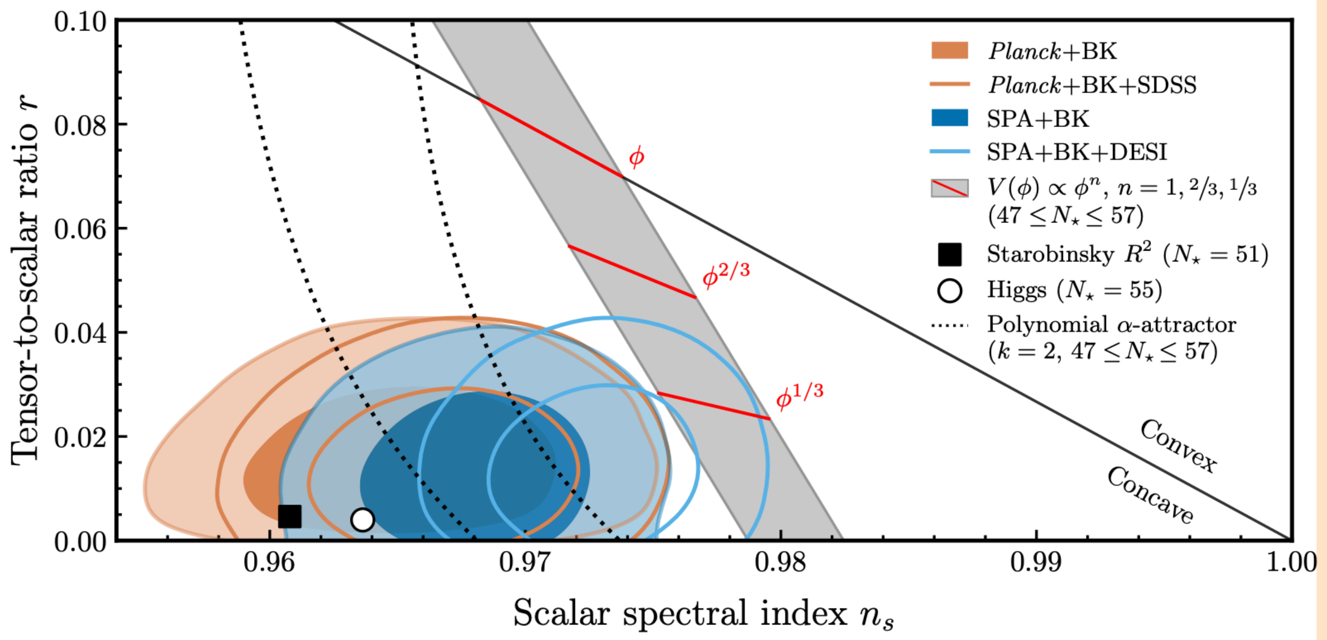

Balkenhol et al 2512.10613

Given the data, one can compare the theory prediction $C_\ell \propto \sum_k T_\ell(k)\mathscr{P}(k)$

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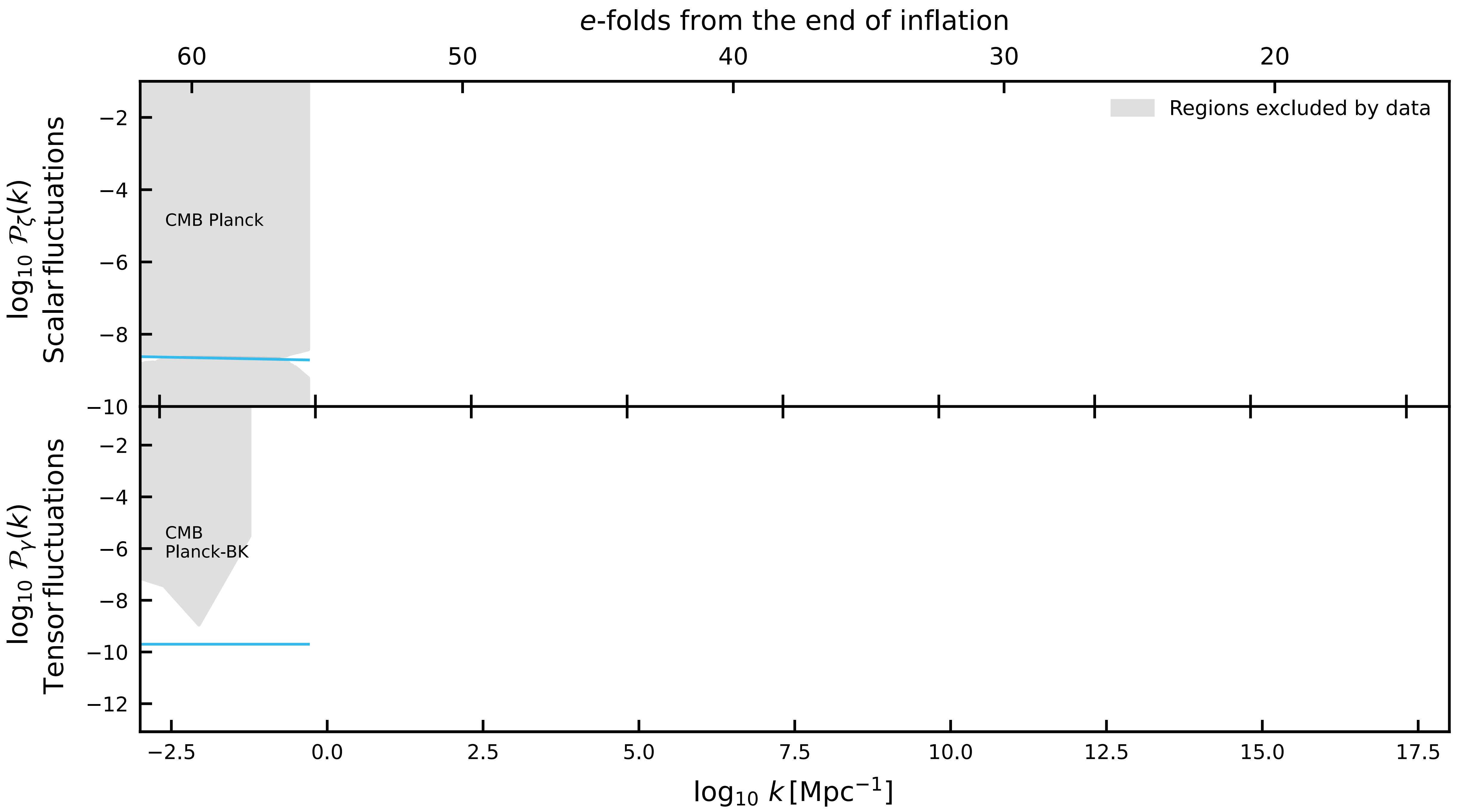
Data are so accurate that we can already rule out models.

CMB is linear, LSS requires non-linear Physics.

A generic prediction of inflation is the production of GW. The search of this smoking gun is the focus of many of the talks in this conference.

Despite some discrepancies between CMB datasets, canonical single field SR is consistent with data. Convex potentials are ruled out.

WHAT WE KNOW ABOUT INFLATION



So far, we have detected only scalar modes.

SF SR is consistent with data, but we have found no smoking gun.

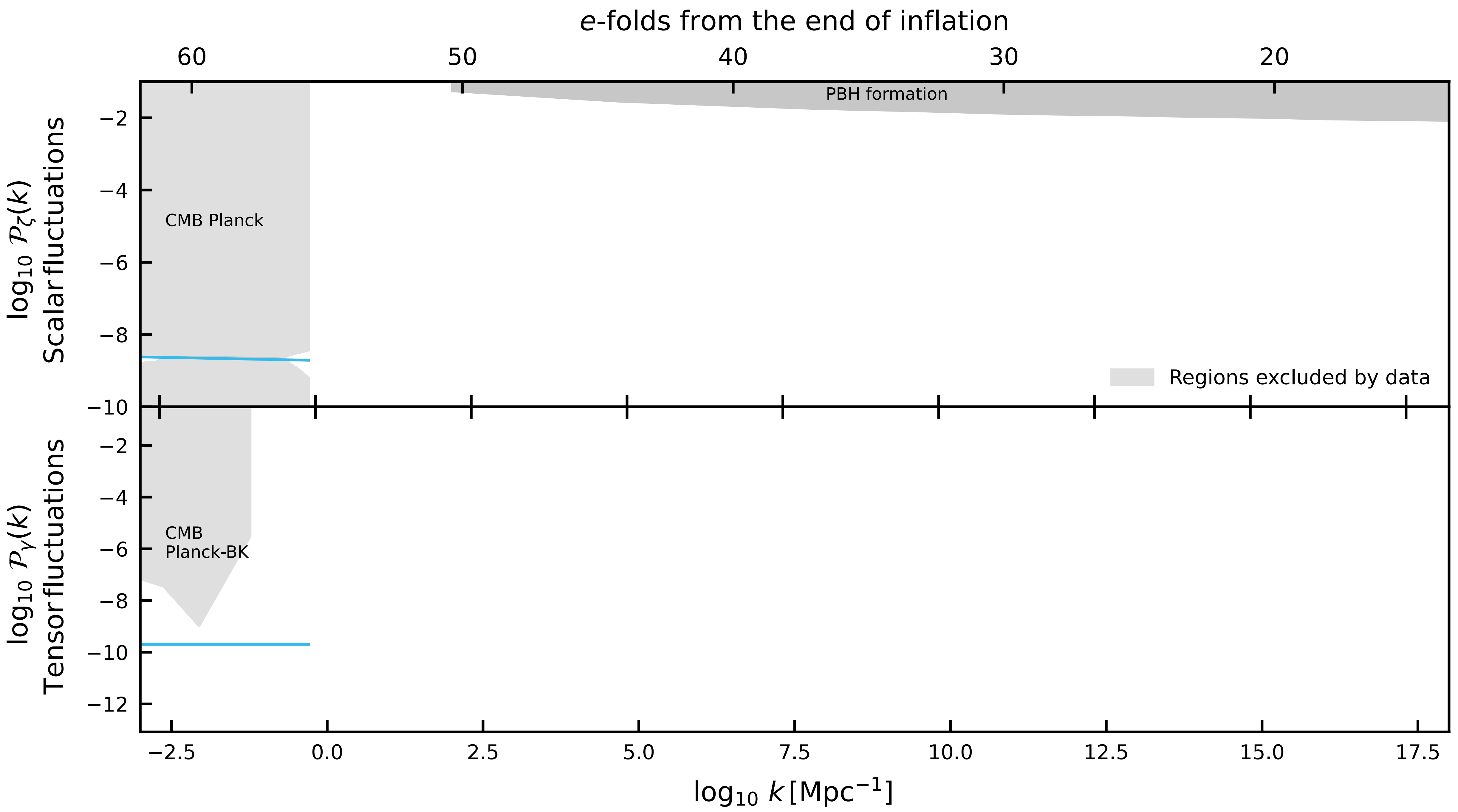
Cosmological observables access only 4-5 $e$folds of the inflationary history.

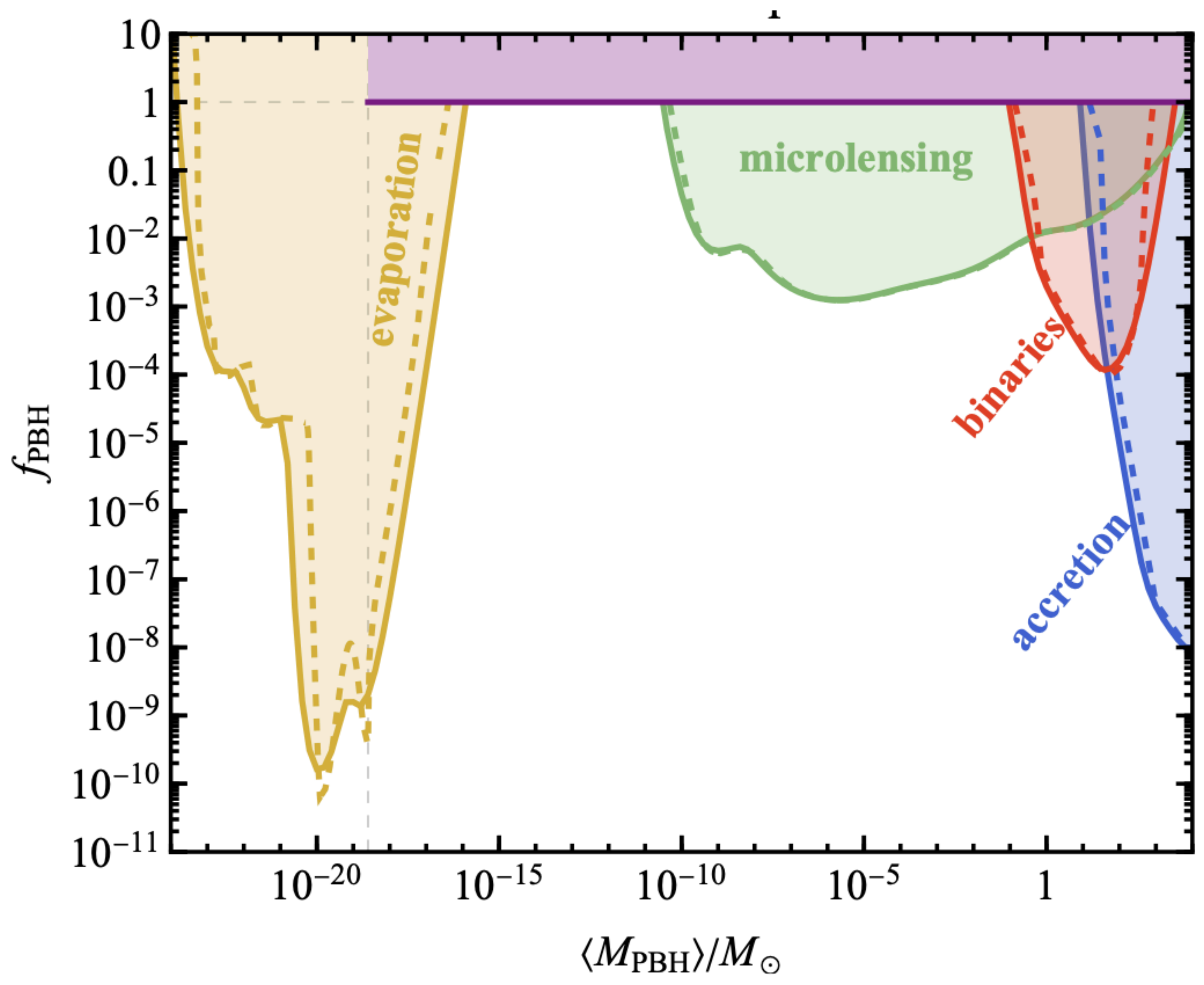
In this talk, I will only discuss inflationary power spectra. Other relevant observables include higher-order correlation functions (PNGs). In fact, this is another generic prediction of inflation, and perhaps another smoking gun.

There are also attempts at considering directly the full distribution of the cosmological perturbations, which encodes more information than the correlation functions.

SMALL-SCALE CONSTRAINTS

PBH OVERPRODUCTION





Iovino et al 2512.13658

Very large perturbations $\delta \gtrsim \delta_c \sim 0.5$ on the scale $f \sim k/2\pi$ collapse into PBHs of mass $M_{\rm PBH} \sim 3 \times 10^{-11} \left(\frac{f}{\rm mHz}\right)^2 M_\odot$ horizon re-entry.

We can set a theoretical constraint on the power spectrum by requiring $f_{\rm PBH}(k) \leq 1$

Constraints are extremely model-dependent, $f_{\rm PBH}(k)$ sensitive to the tail of $P(\zeta)$.

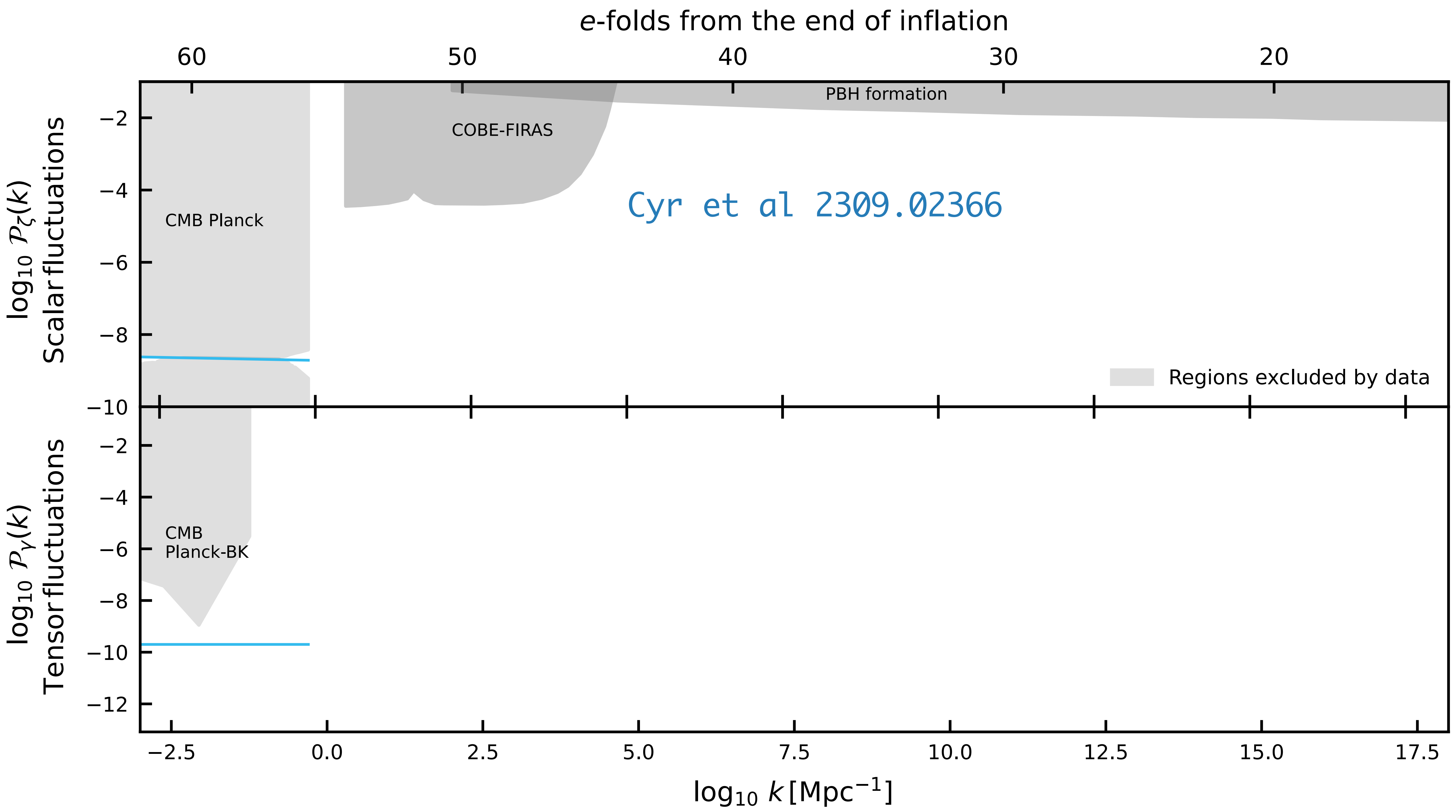
See e.g. Franciolini et al 2306.17149

Factor of ~ 2 uncertainty in the methodology to compute the abundance. See e.g. Cecchini et al 2503.10805.

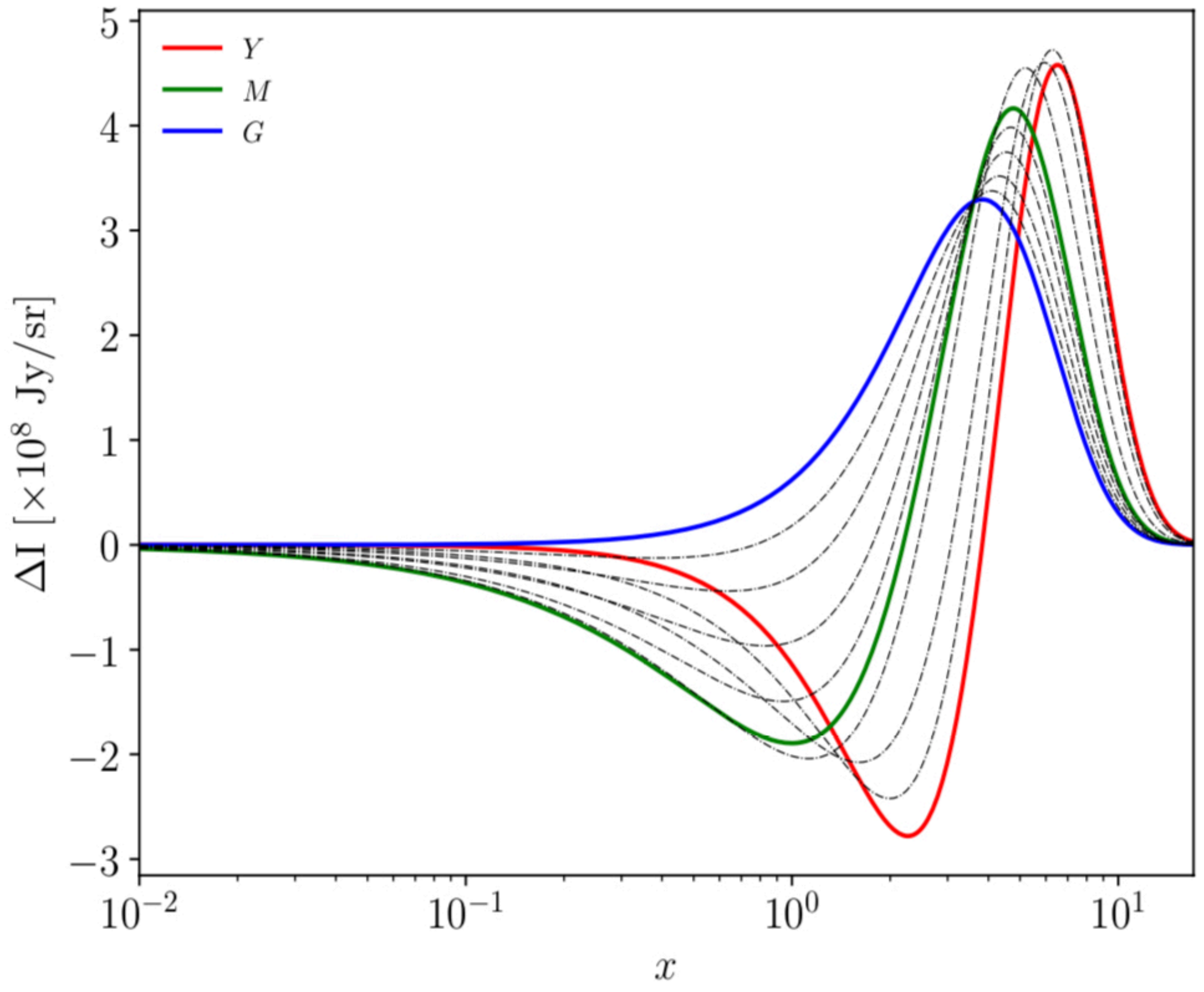
While translating these constraints on the primordial power spectrum is tricky, the abundance of PBHs is tightly constrained. There is only a small window at $\sim M_{\rm PBH} \sim 10^{-12} M_\odot$ where they can constitute the totality of dark matter

It is also possible to form PBHs with alternative mechanisms that do not involve the collapse of large fluctuations.

SPECTRAL DISTORTIONS



Cyr et al 2309.02366



Lucca et al 1910.04619

Silk damping produces distortions in the CMB spectrum.

Window functions are very wide, weakly sensitive to the shape of spectrum.

Also sensitive to GWs. Constraints from FIRAS are outside the plot limits.

They are technically ‘cosmo’ constraints, so I shouldn’t be discussing them…

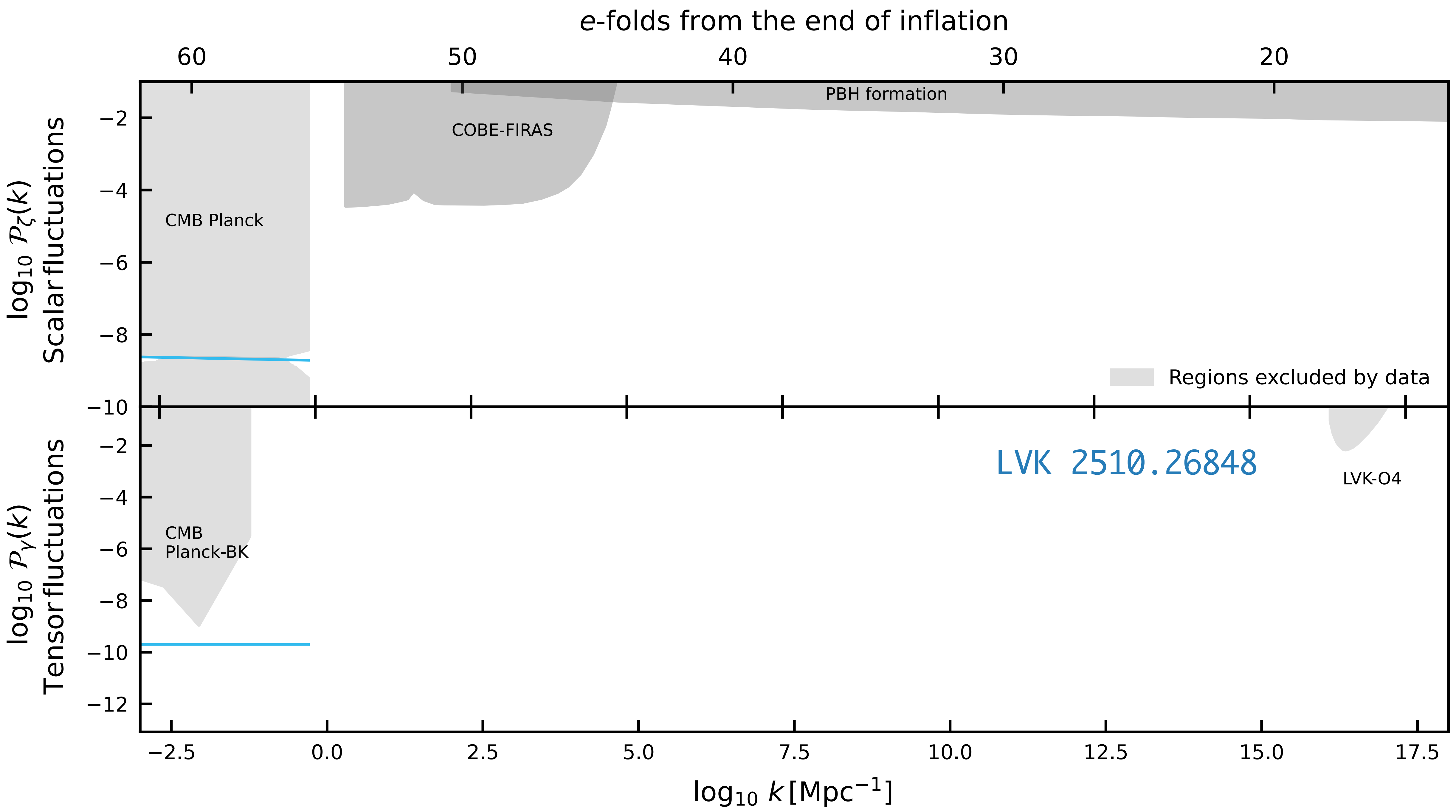
Sensitive to Non-Gaussianities of the curvature perturbations.

Sharma et al 2404.18475
Pritchard et al 2505.08442

μT and $\mu\mu$ can also be used to learn about PNGs such as $f_{\mathrm{NL}}^{\mathrm{loc}}$ or τ_{NL}

Pajer & Zaldarriaga 1201.5375
Bianchini & Fabbian 2206.0276

LIGO-VIRGO-KAGRA



LVK has not detected a gravitational wave background (yet).

We know how to translate constraints on the strain $\Omega_{\rm GW}(f)$ into constraints on $\mathscr{P}_\gamma(k)$ through the relation $\Omega_{\rm GW}(f) = T_{\rm GW}(f)\,\mathscr{P}_\gamma(f)$

LVK has not observed $\Omega_{\rm GW}$. We can translate this into an upper bound on $\mathscr{P}_\gamma(k)$.

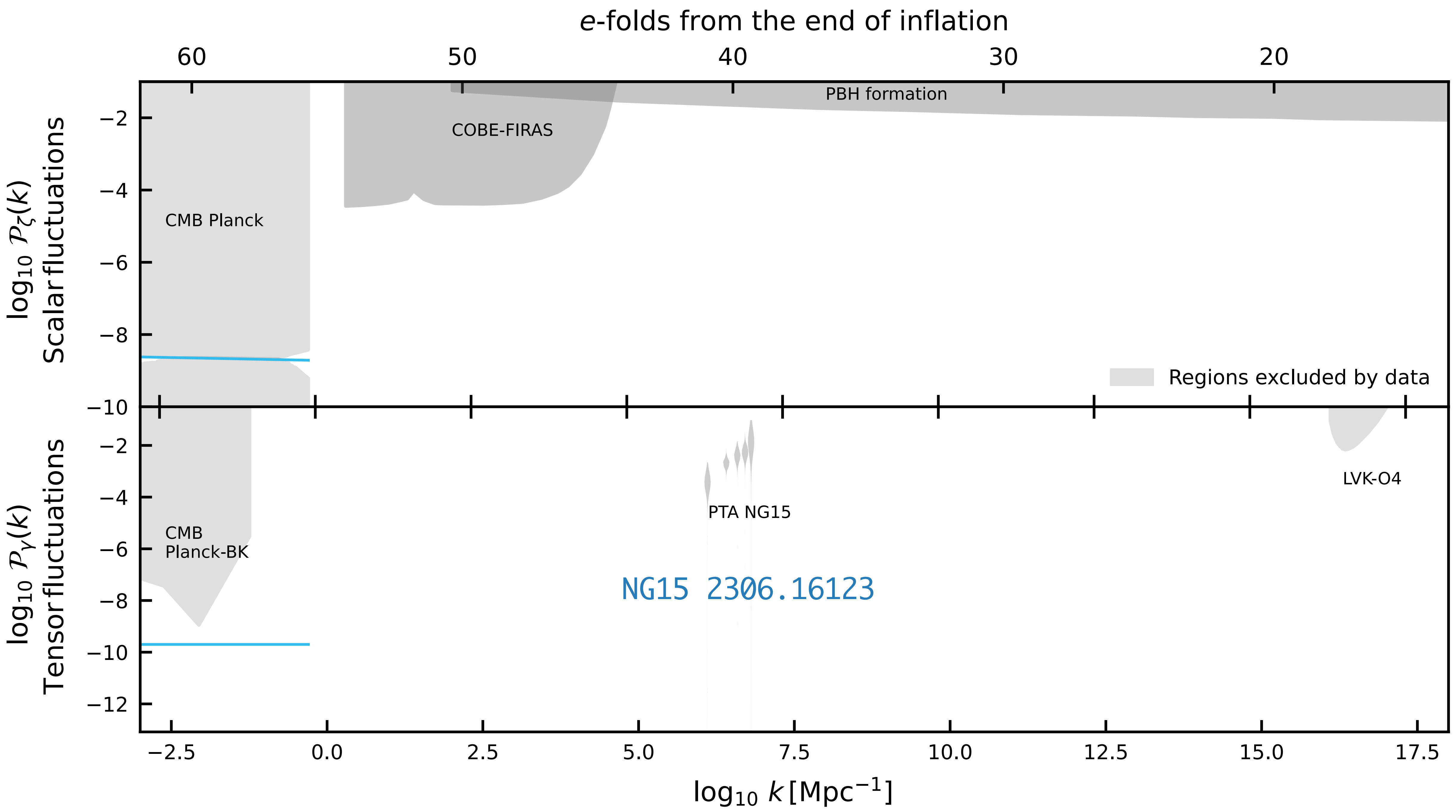
The GWB acts as an extra source of noise on each detector. It is detected through cross-correlation of independent detectors.

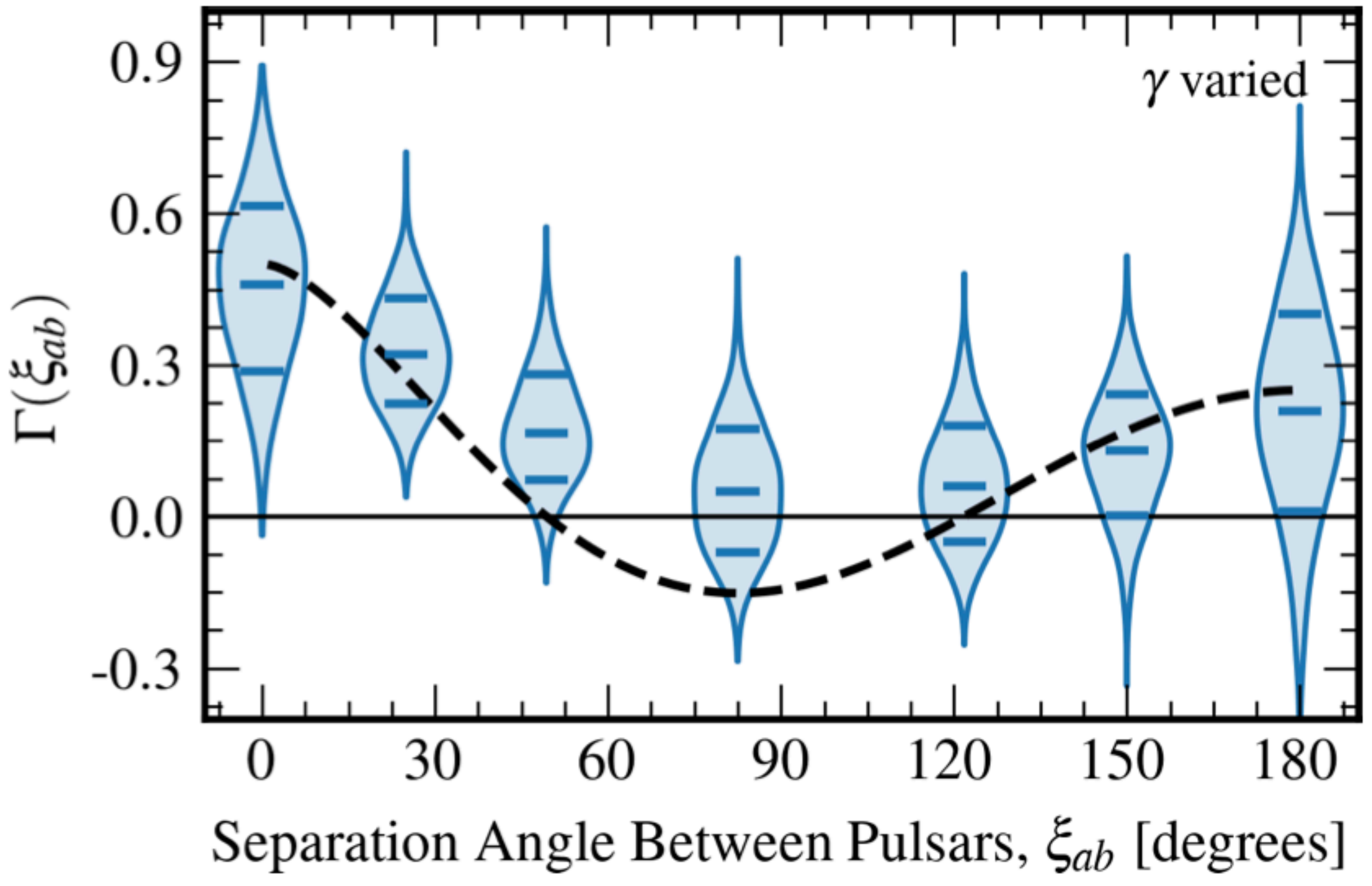
See e.g. Maggiore 2007

Non-standard thermal histories, such as early matter domination, kination, EDE etc., can modify this calculation. We (mostly) know how to take these effects into account, but they could be degenerate with the inflationary production mechanism.

See e.g. Kuroyanagi et al 1301.1778

PULSAR TIMING ARRAYS





In June 2023, major PTA collaborations detected a signal compatible with a GWB.

The correlations follow the Hellings–Downs pattern expected for a GWB.

The constraints plotted are from NG15. Other PTA collaborations like EPTA and PPTA found consistent results.

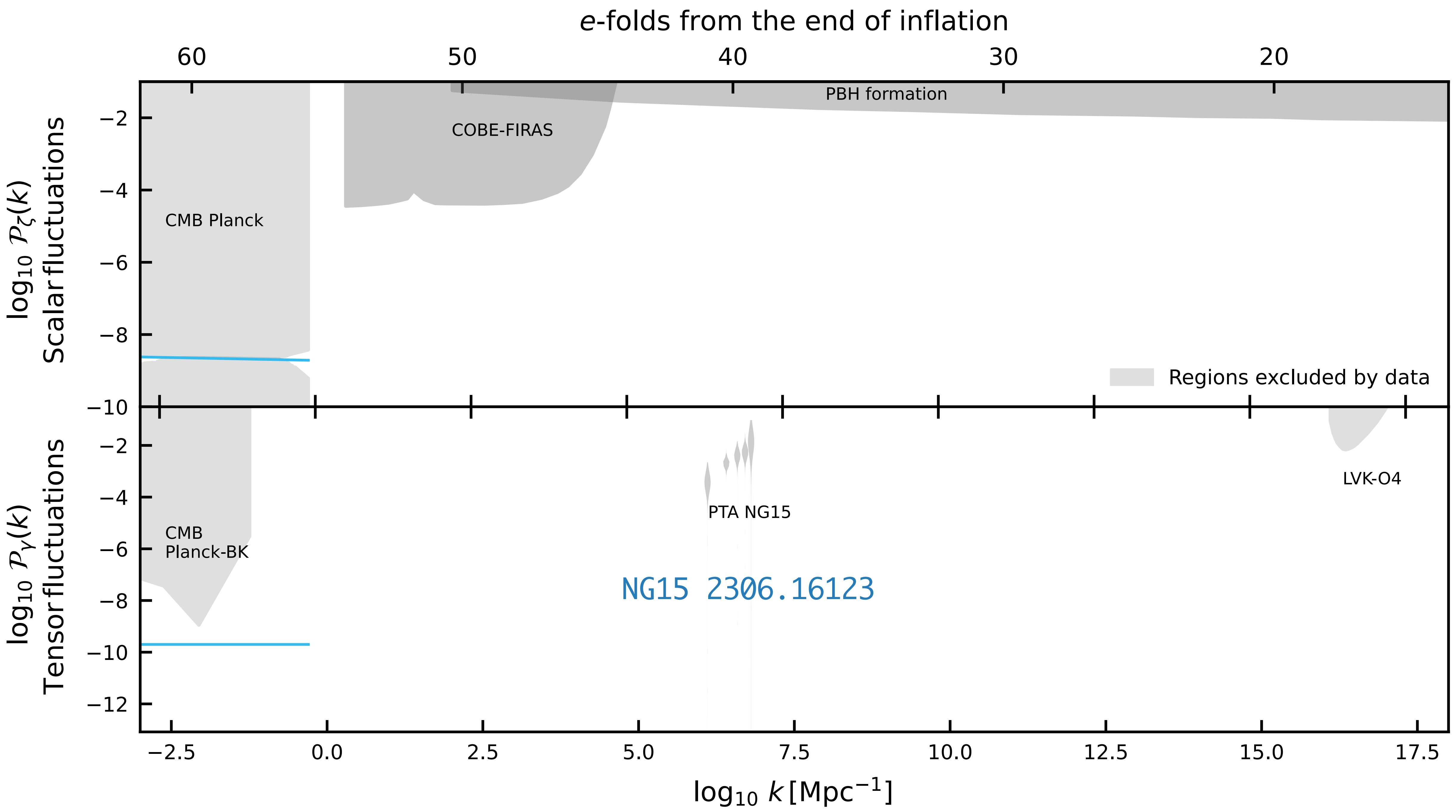
EPTA 2301.16124

PPTA 2306.16215

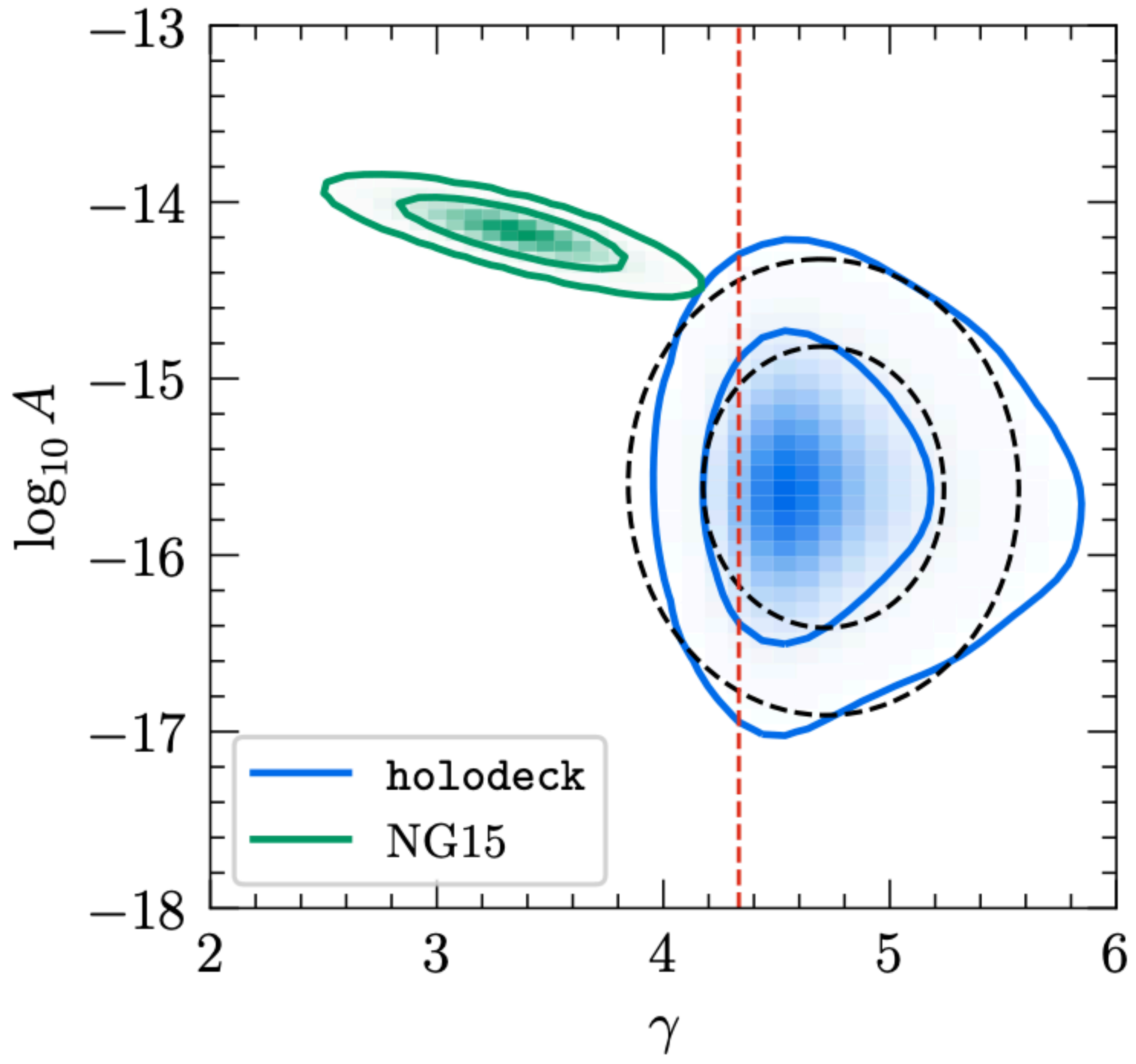
Earlier releases showed hints of a possible signal, but without statistically significant evidence for the Hellings–Downs correlation.

NG12.5 2009.04496

PULSAR TIMING ARRAYS



NG15 2306.16123



NG15 2306.16129

In June 2023, major PTA collaborations detected a signal compatible with a GWB.

The correlations follow the Hellings–Downs pattern expected for a GWB.

Strikingly, the signal appears in mild tension with standard astrophysical models of inspiraling SMBHs.

The constraints plotted are from NG15. Other PTA collaborations like EPTA and PPTA found consistent results.

EPTA 2301.16124
PPTA 2306.16215

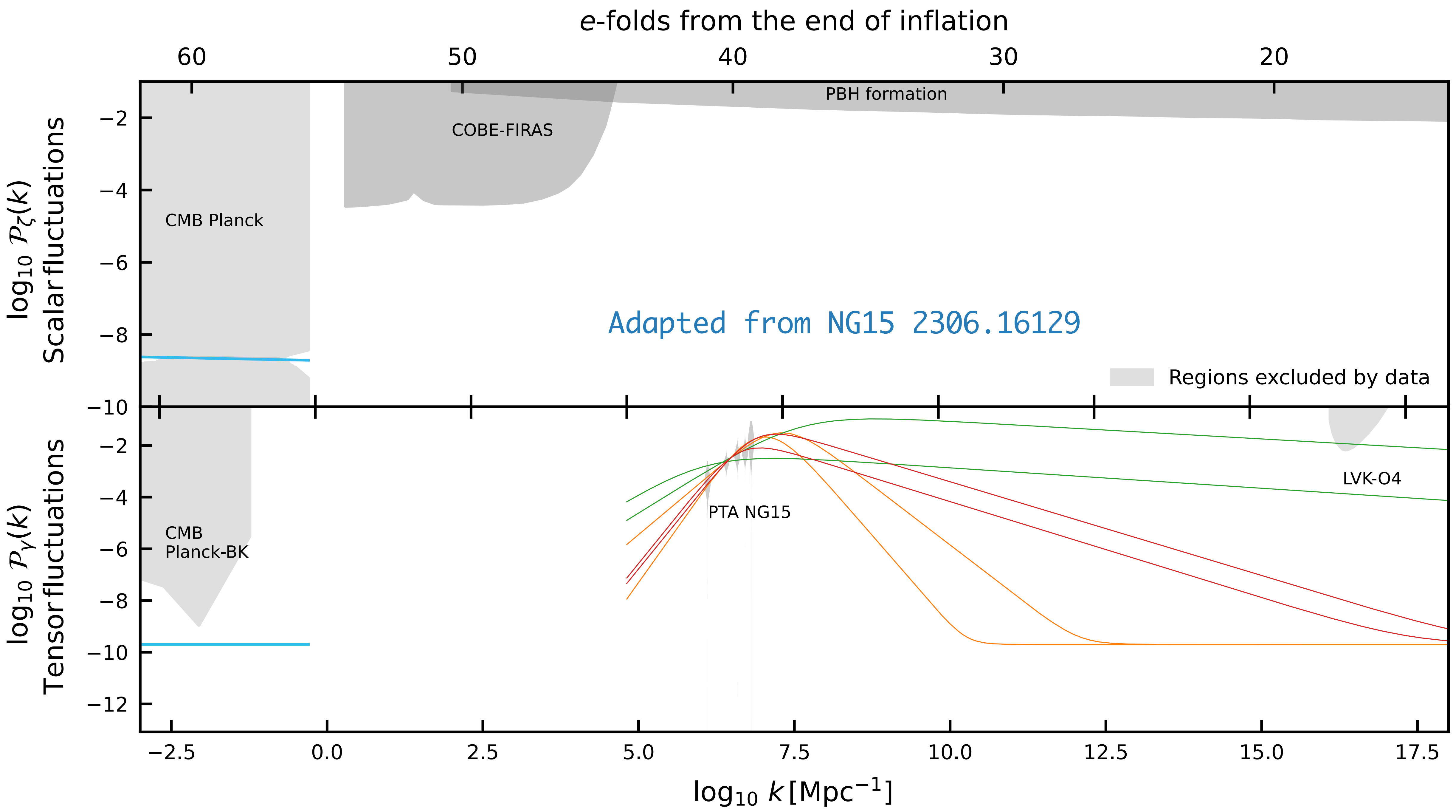
Earlier releases showed hints of a possible signal, but without statistically significant evidence for the Hellings–Downs correlation.

NG12.5 2009.04496

Environmental effects, finite number of sources effect etc change slightly the result.

NG15 2306.16220, Sato-Polito & Zaldarriaga 2312.06756

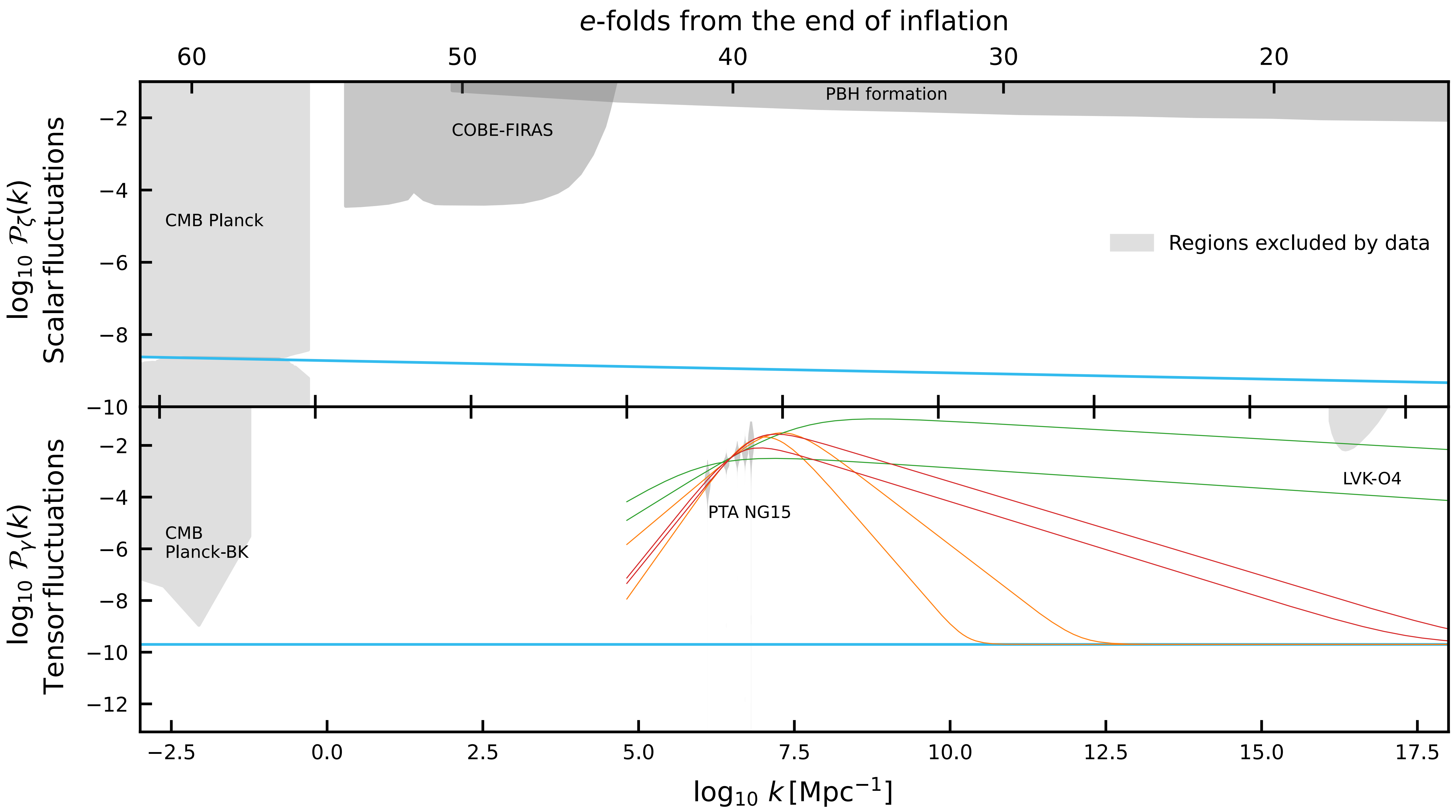
PULSAR TIMING ARRAYS



Taking this tension at face value, many models appear to fit the data better than SMBHs, including Phase Transitions, Cosmic Strings, Domain Walls etc

The plots are digitized and extrapolated far away from nHZ, where the shape may be slightly inaccurate.

PULSAR TIMING ARRAYS



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…unless one invokes a very non-standard thermal history after inflation.

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The GWB from SFSR lies orders of magnitude below current experiments…

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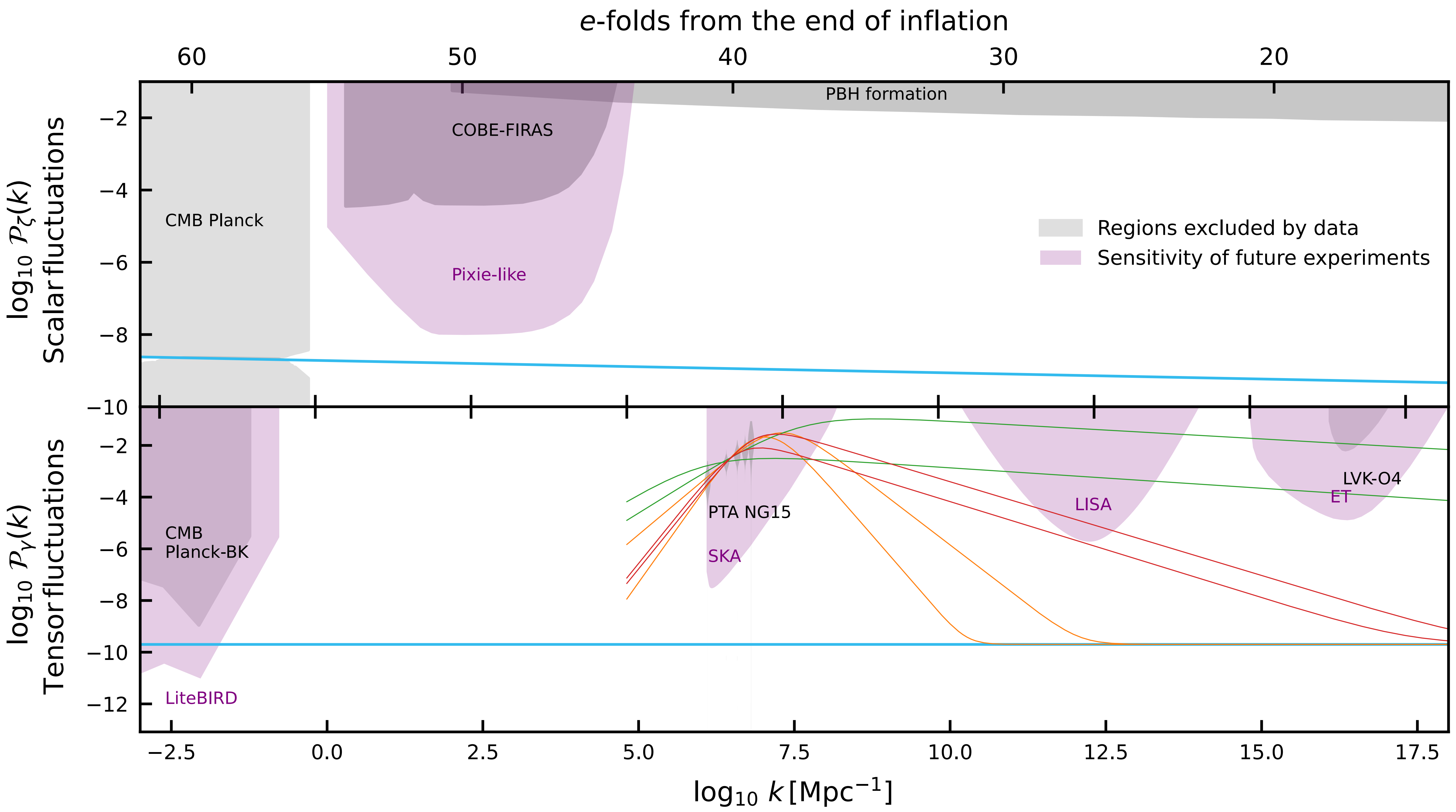


Taking this tension at face value, many models appear to fit the data better than SMBHs, including Phase Transitions, Cosmic Strings, Domain Walls etc

The GWB from SFSR lies orders of magnitude below current experiments…

…and the situation won't get much better in the foreseeable future.

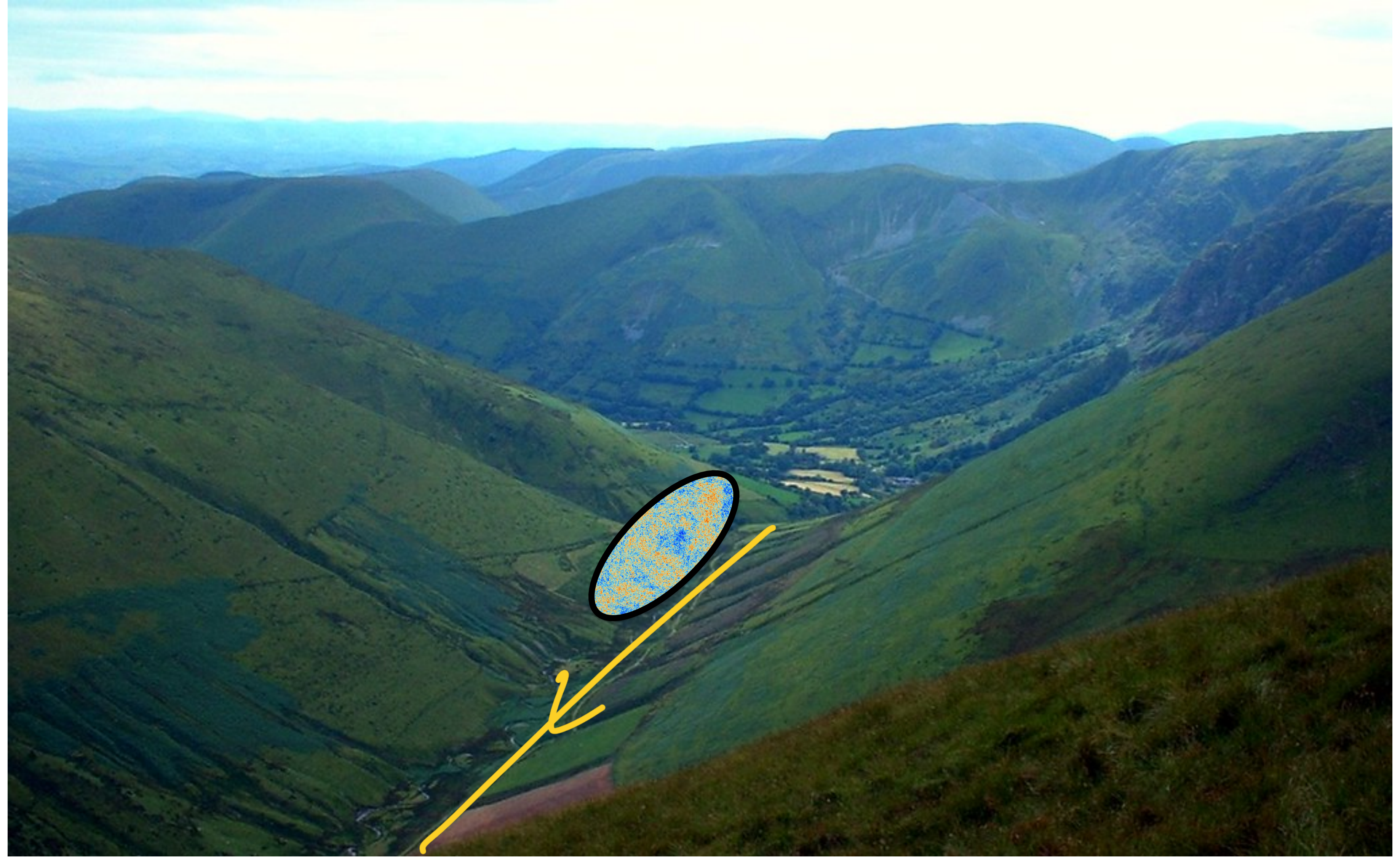
The plots are digitized and extrapolated far away from nHZ, where the shape may be slightly inaccurate.

…unless one invokes a very non-standard thermal history after inflation.

Highly futuristic missions such as DECIGO and BBO could achieve sensitivities comparable to LiteBIRD, but on much longer timescales—if realized.

BUT WHAT SHOULD WE EXPECT?

SR for 60 $e$folds



SR for a few $e$folds

Cosmological data are consistent with SF SR, but test only 4-5 $e$folds.

What if the SR is temporarily violated during the 60 $e$folds?

We can expect strong scale-dependences, and amplification of the power spectra at small scales.

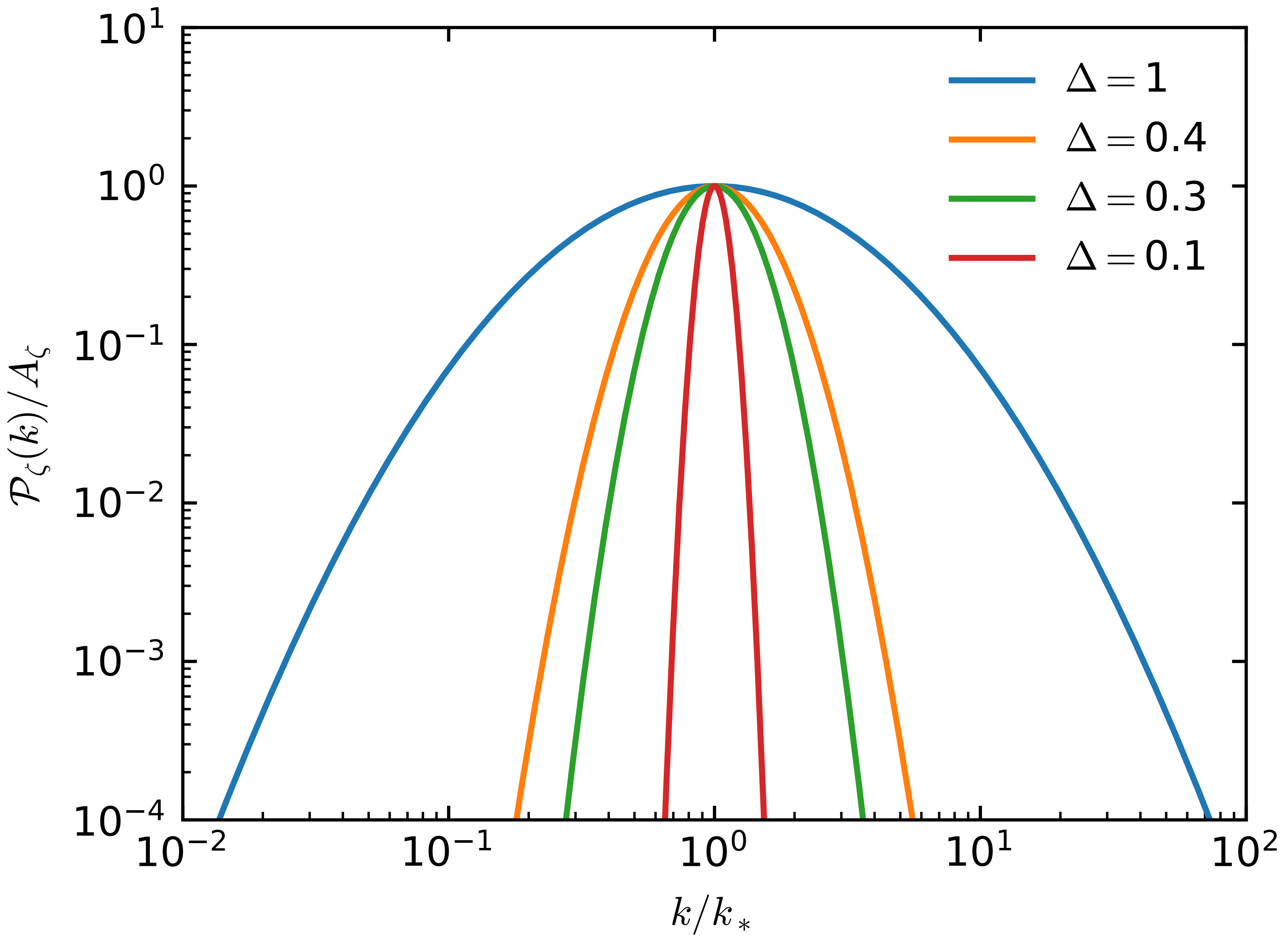
Many UV-complete embeddings of inflation predict the presence of additional fields. In this broader landscape, single-field SR inflation represents a tuned realization.

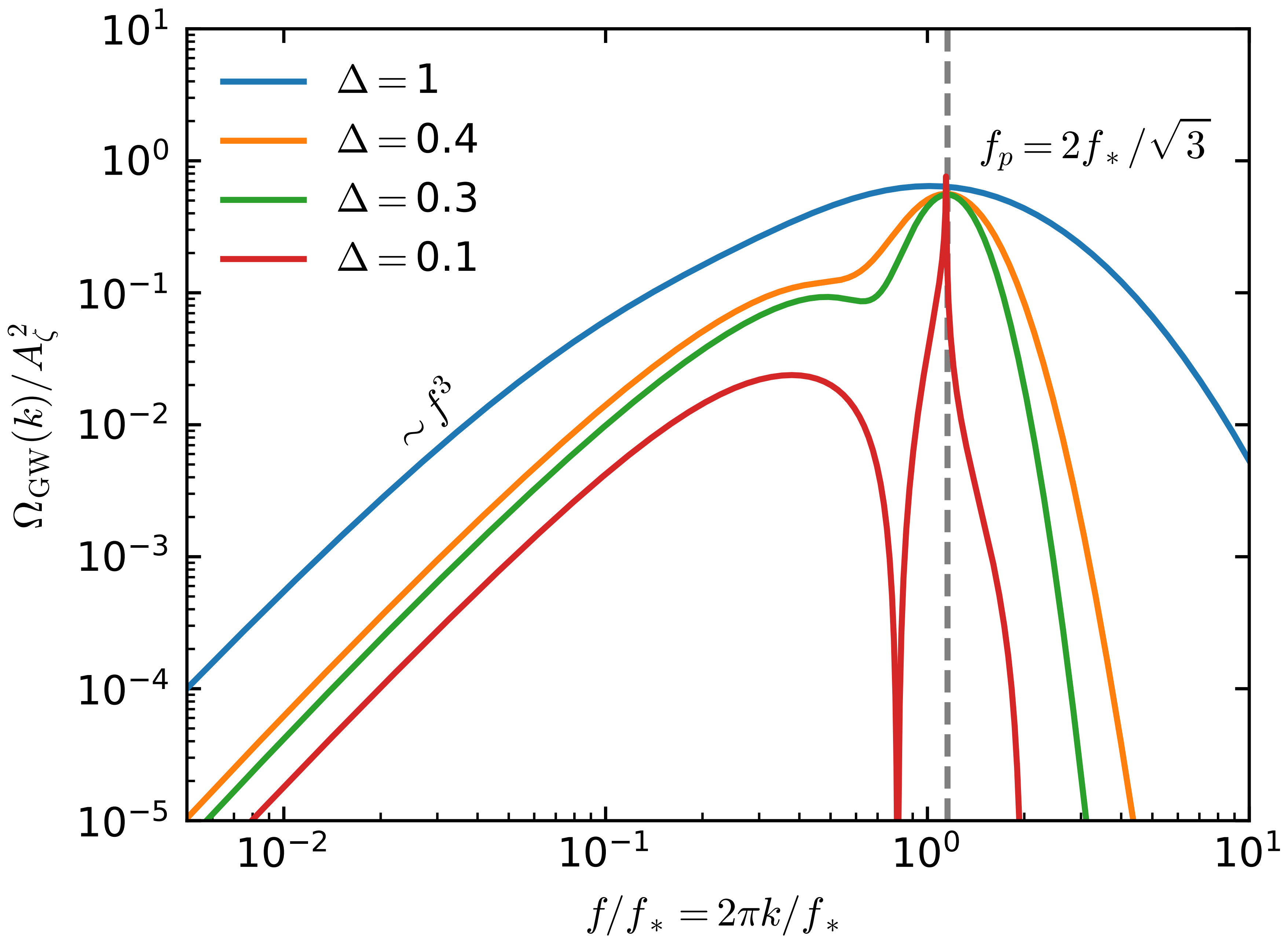
The trajectory may feature turns, bumps, or steps, which can excite oscillations of massive degrees of freedom.

Chen 1002.1416
Achucarro et al 2203.08128

SCALAR-INDUCED GWS

SCALAR-INDUCED GWS





Large scalar perturbations source gravitational waves at 2nd order in perturbation theory when they re-enter the horizon during radiation era.

The GWB exhibits interesting features, notably a double-peaked structure if $\mathscr{P}_\gamma(k)$ sufficiently narrow.

The GW background is obtained by convolving the power spectrum of scalars with a known kernel:

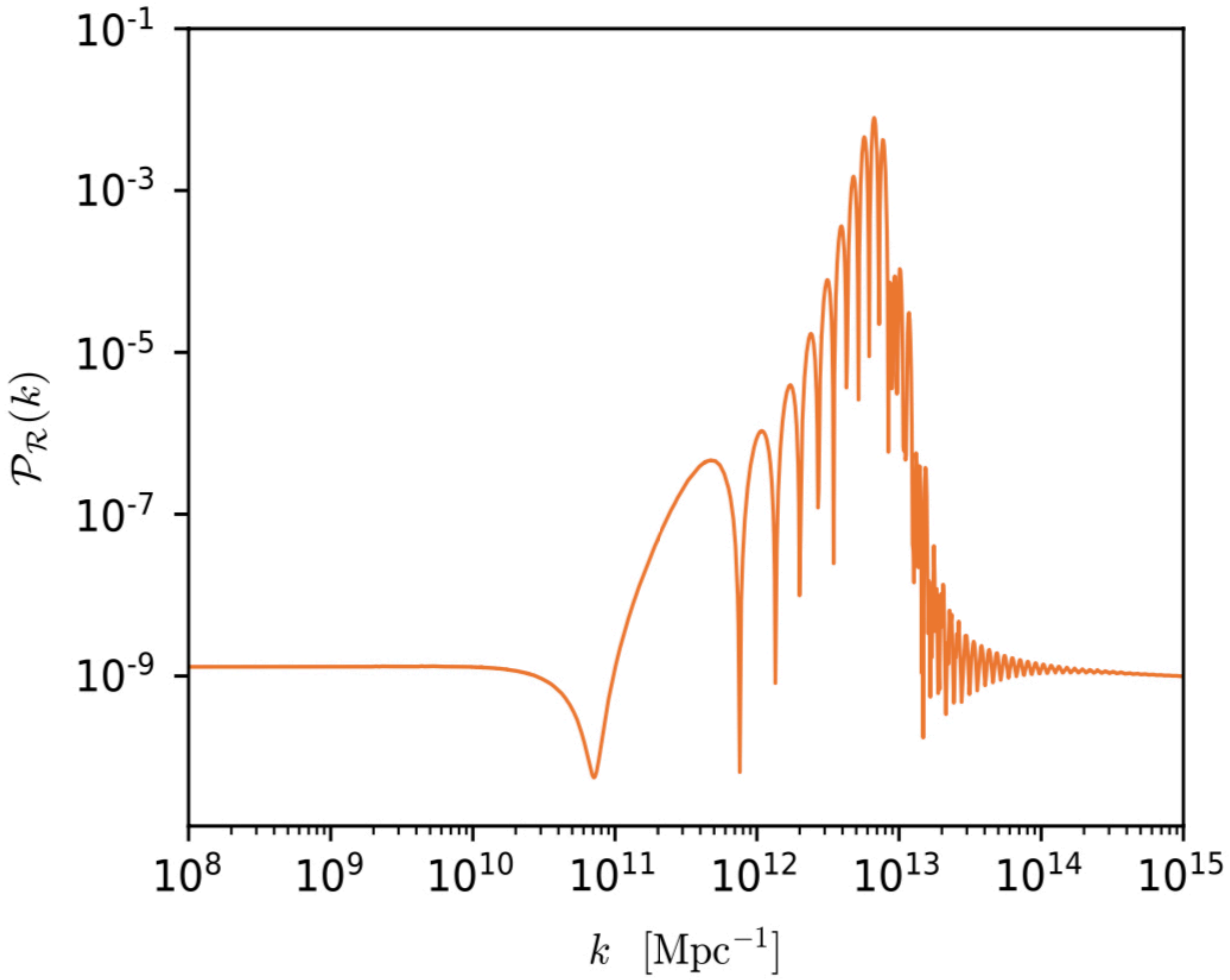
$$\Omega(k) \sim \int dudv\, \mathscr{P}_\zeta\left(uk\right) \mathscr{P}_\zeta\left(vk\right) K(u,v)$$

Matarrese et al astro-ph/9707278
Ananda et al gr-qc/0612013

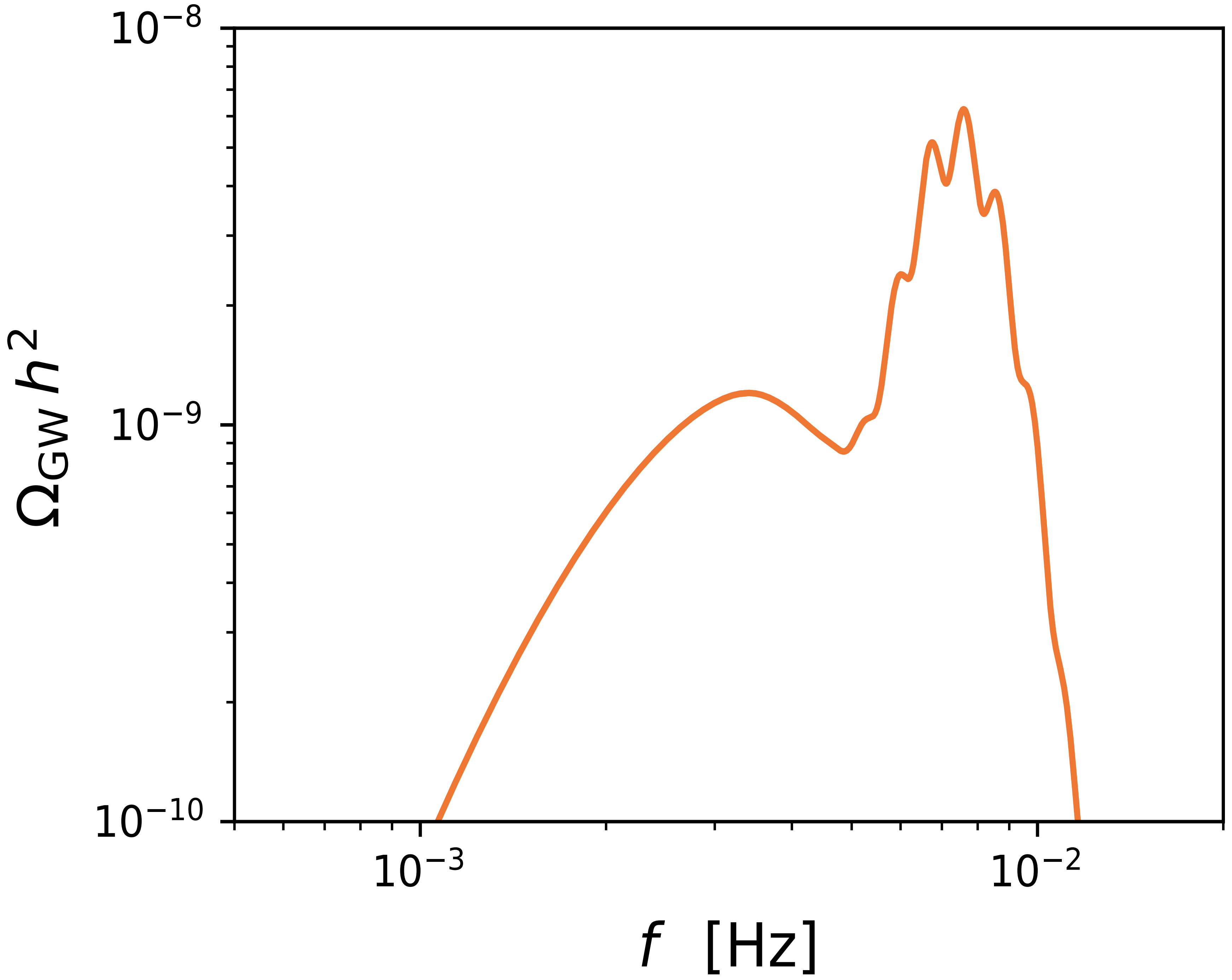
The kernel is actually sensitive to the thermal history at horizon re-entry. Here I focus on radiation domination.

Domènech et al 2005.12314

SCALAR-INDUCED GWS



Braglia, Chen, Hazra 2012.05821



Large scalar perturbations source gravitational waves at 2nd order in perturbation theory when they re-enter the horizon during radiation era.

The GWB exhibits interesting features, notably a double-peaked structure if $\mathscr{P}_\gamma(k)$ sufficiently narrow.

Interesting oscillatory features can also be produced in certain scenarios.

The GW background is obtained by convolving the power spectrum of scalars with a known kernel:

$$\Omega(k) \sim \int du dv\, \mathscr{P}_\zeta\left(uk\right) \mathscr{P}_\zeta\left(vk\right) K(u,v)$$

Matarrese et al astro-ph/9707278
Ananda et al gr-qc/0612013

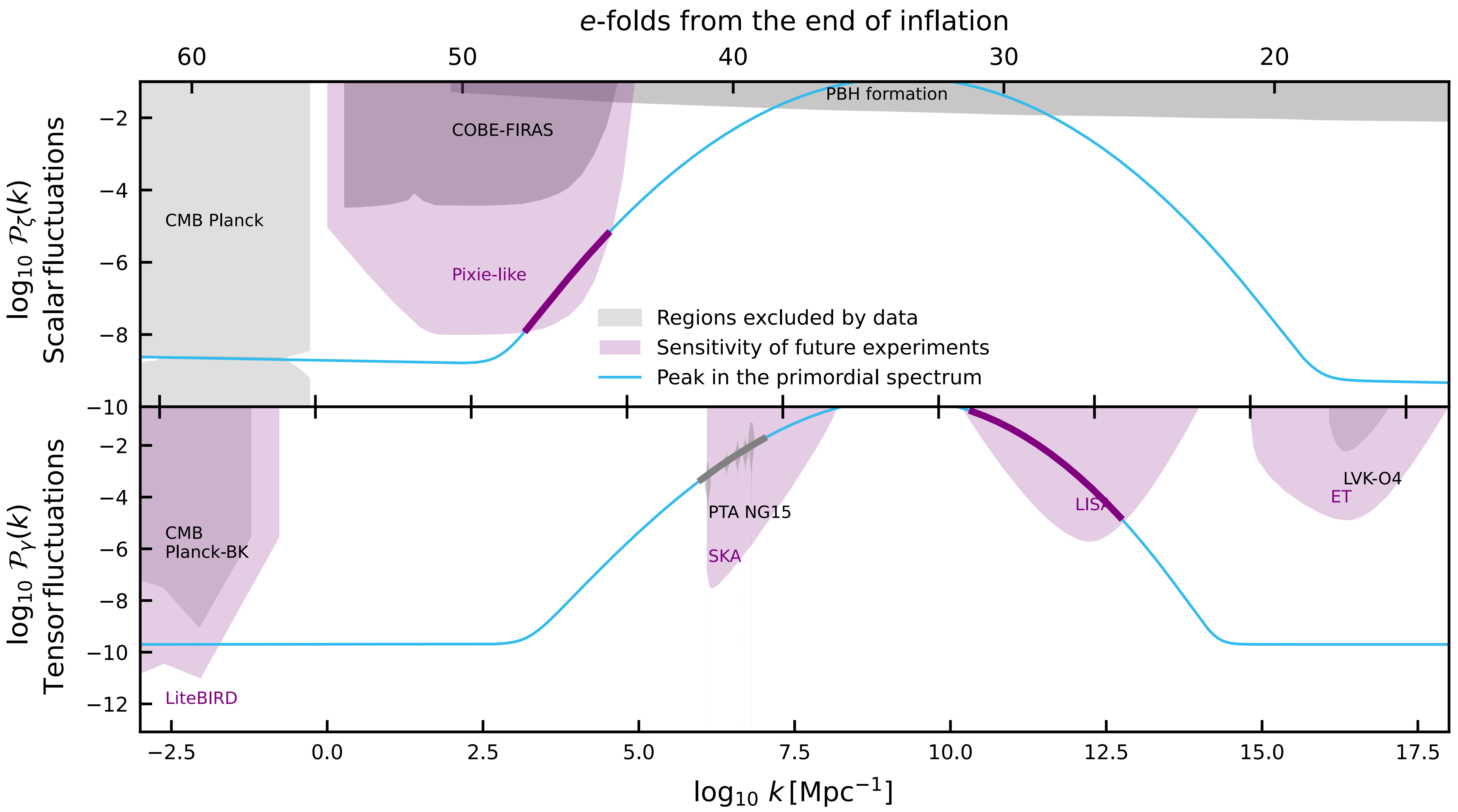
The kernel is actually sensitive to the thermal history at horizon re-entry. Here I focus on radiation domination.

Domènech et al 2005.12314

PNGs of the curvature perturbation can also change the spectral shape around the secondary peak.

Adshead et al 2105.01659

SCALAR-INDUCED: PTA



SIGWs provide one of the best fits to PTA data.

NG15 2306.16129, Figueroa et al 2307.02399, Ellis et al 2308.08546

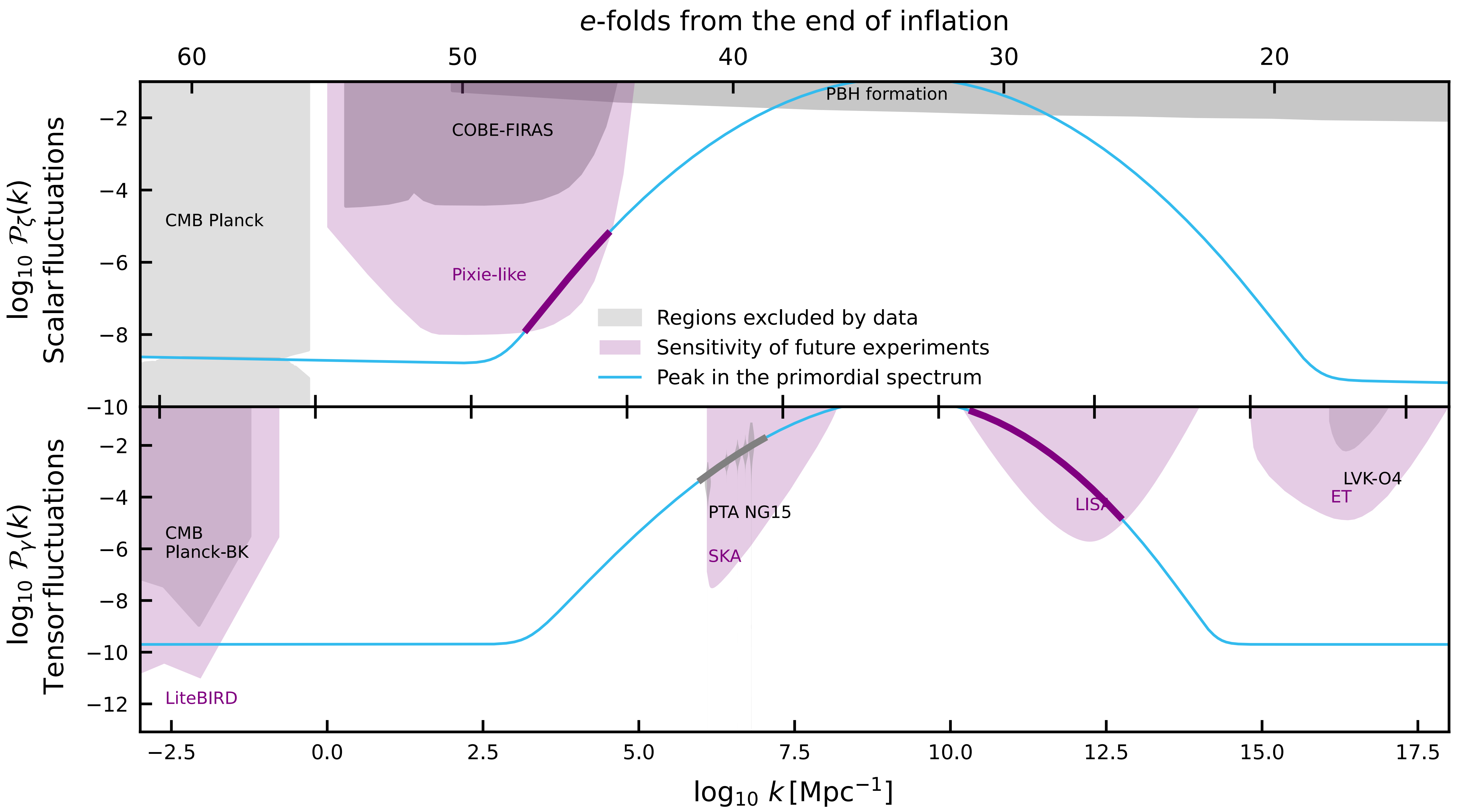
The amplitude seem to be in conflict with constraints from PBH overproduction.

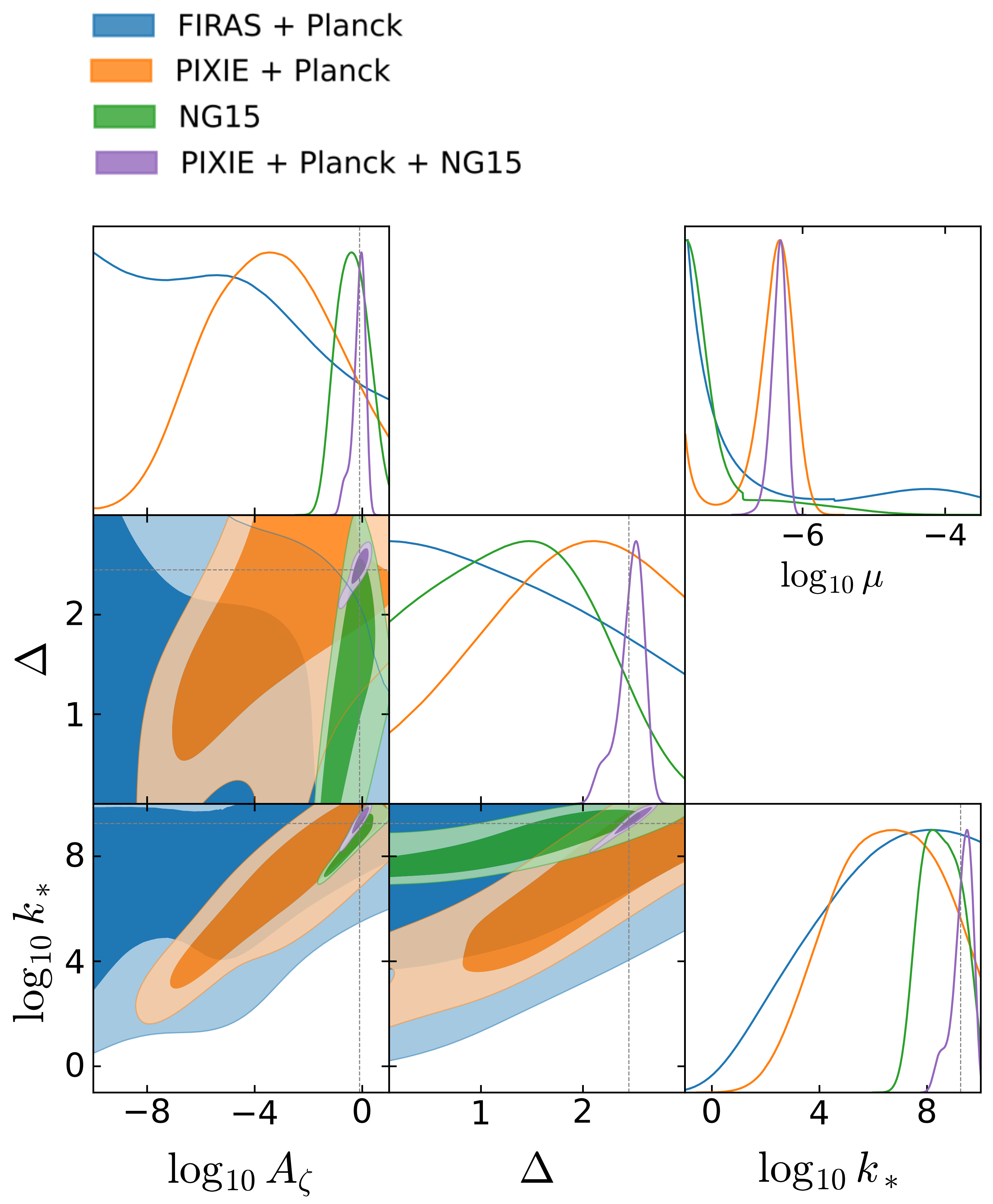
The part of the signal which fits the PTA data is at frequencies smaller than the peak.

If the primordial perturbations are highly non-Gaussian, or the EoS at horizon re-entry is not $w = 1/3$, PBH constraints are evaded.

Franciolini et al 2306.17149, Balaji et al 2307.08552

SCALAR-INDUCED: PTA





Tagliazucchi, Braglia, Finelli, Pieroni 2310.08527

SIGWs provide one of the best fits to PTA data.

NG15 2306.16129, Figueroa et al 2307.02399, Ellis et al 2308.08546

The amplitude seem to be in conflict with constraints from PBH overproduction.

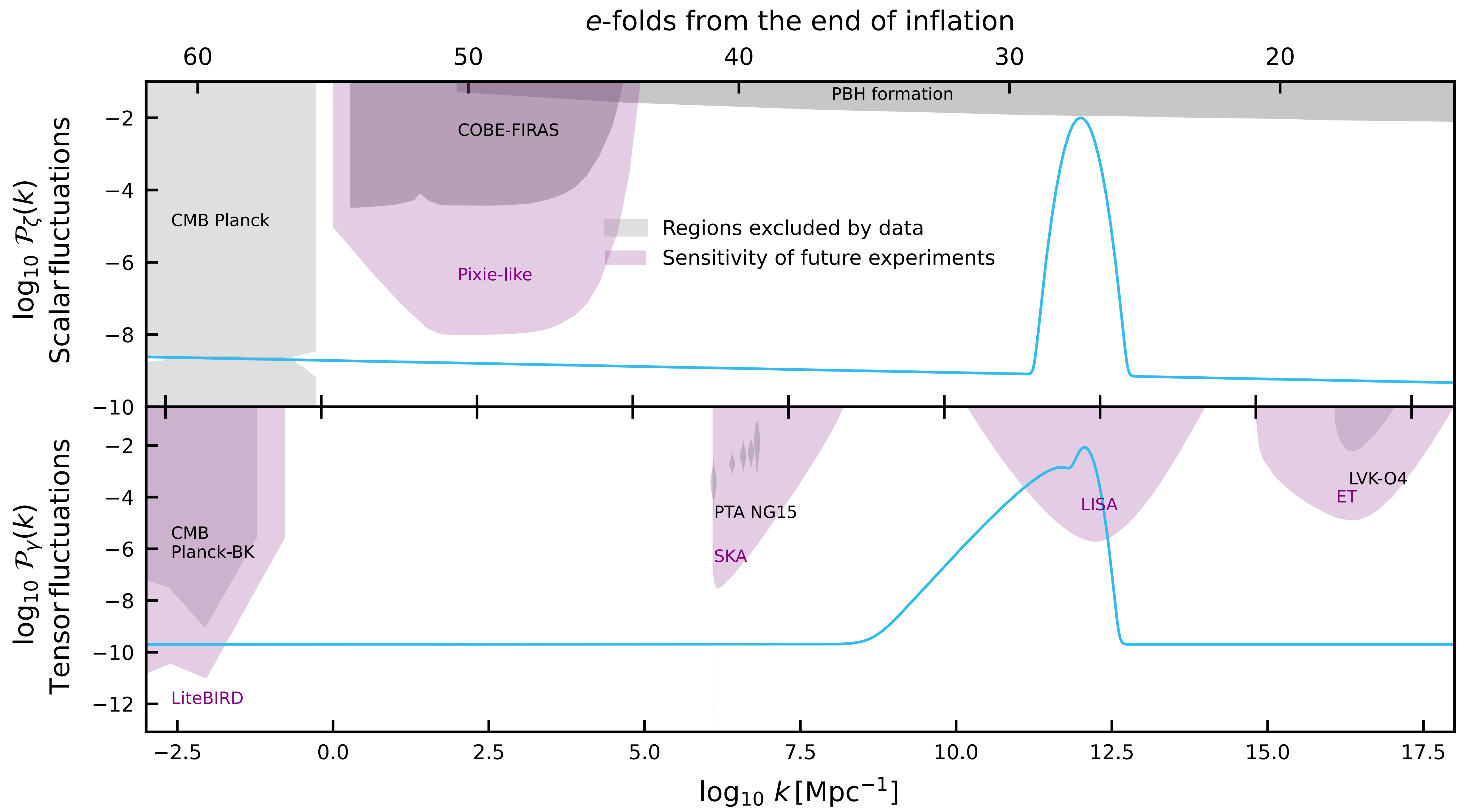
This will inevitably produce a SD signal detectable by future experiments such as FOSSIL, which has the potential to confirm or rule out the SIGW hypothesis.

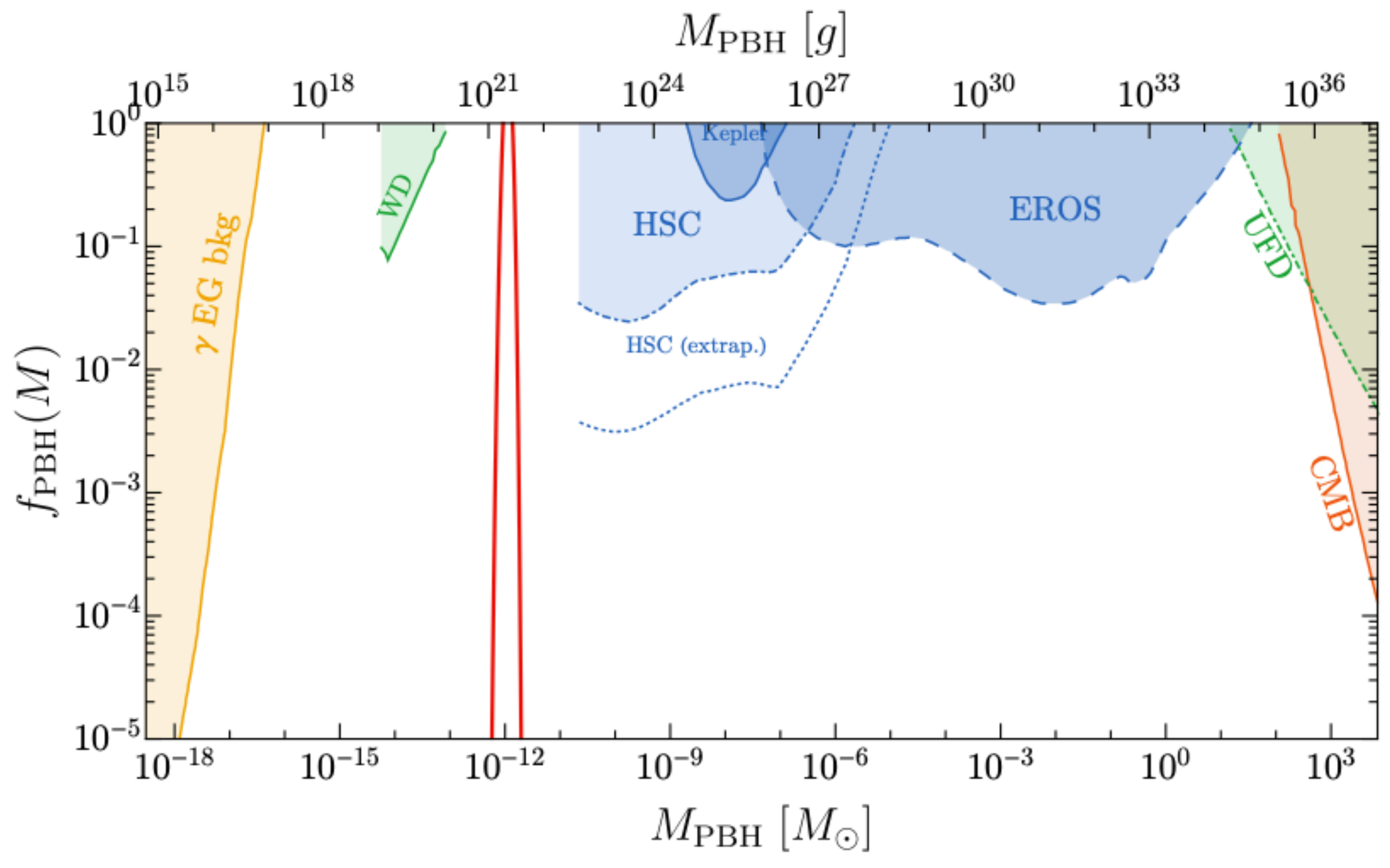
The part of the signal which fits the PTA data is at frequencies smaller than the peak.

If the primordial perturbations are highly non-Gaussian, or the EoS at horizon re-entry is not $w = 1/3$, PBH constraints are evaded.

Franciolini et al 2306.17149, Balaji et al 2307.08552

SCALAR-INDUCED: LISA





Bartolo et al 1810.12218

Remember the relation between PBH mass and frequency of the peak in the PPS:

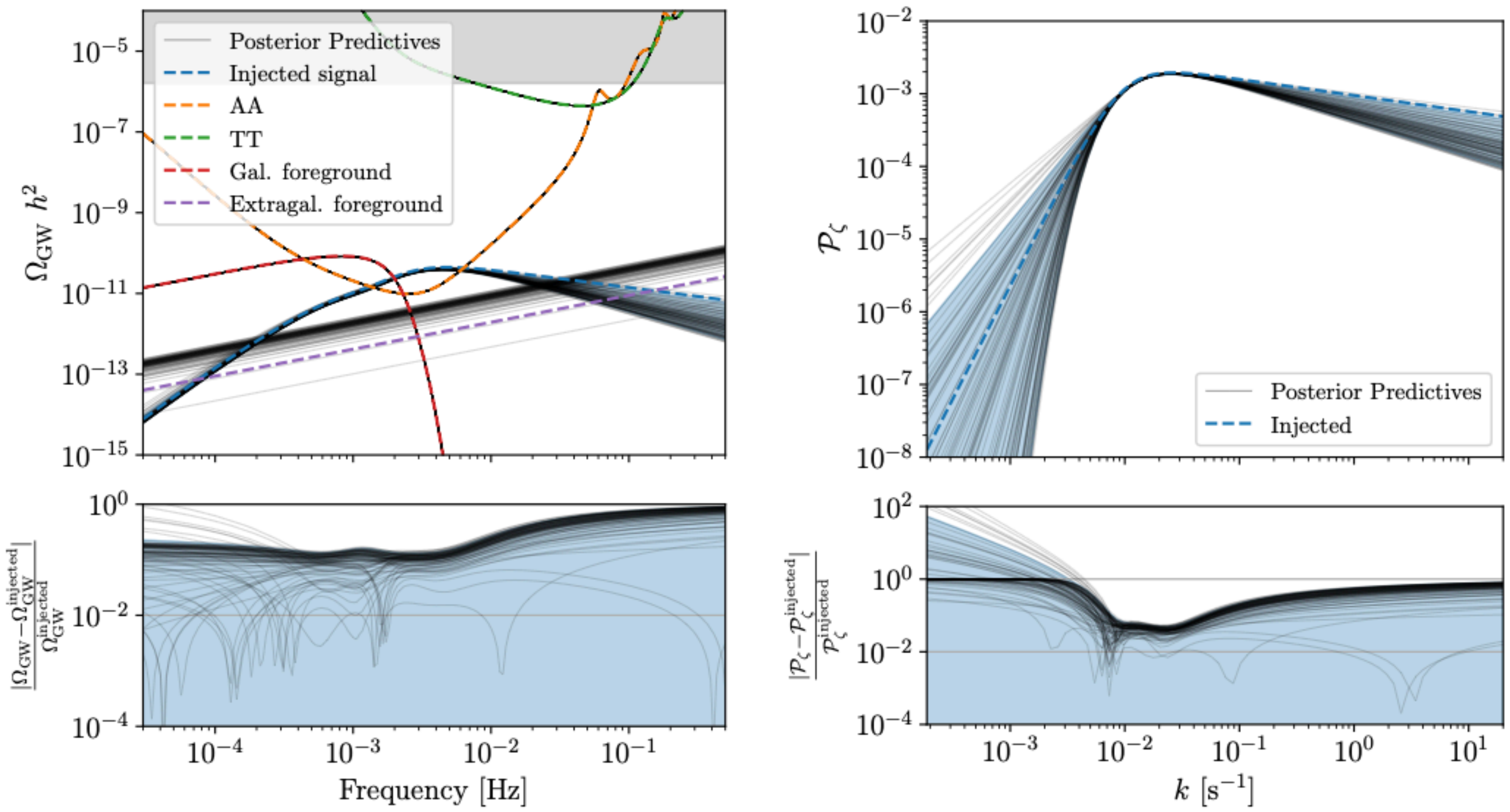
$$M_{\rm PBH} \sim 3 \times 10^{-11} \left(\frac{f}{\rm mHz}\right)^2 M_\odot$$

A spectrum peaked in the mHz, where LISA has maximum sensitivity produces asteroid mass PBHs, which can be 100% of the CDM.

With the LISA CosWG we explored the capability of LISA to detect the SIGW signal produced by the perturbations collapsing into PBHs.

Braglia et al 2407.04356 El Gammal et al 2501.11320

SCALAR-INDUCED: LISA



We developed a pipeline to reconstruct the primordial scalar power spectrum from $\Omega_{\rm GW}(f)$, enabling both template-based and free-form analyses.

LISA has the potential to place accurate constraints on the scalar power spectrum, either through a detection of the GWB

Braglia et al 2407.04356 El Gammal et al 2501.11320

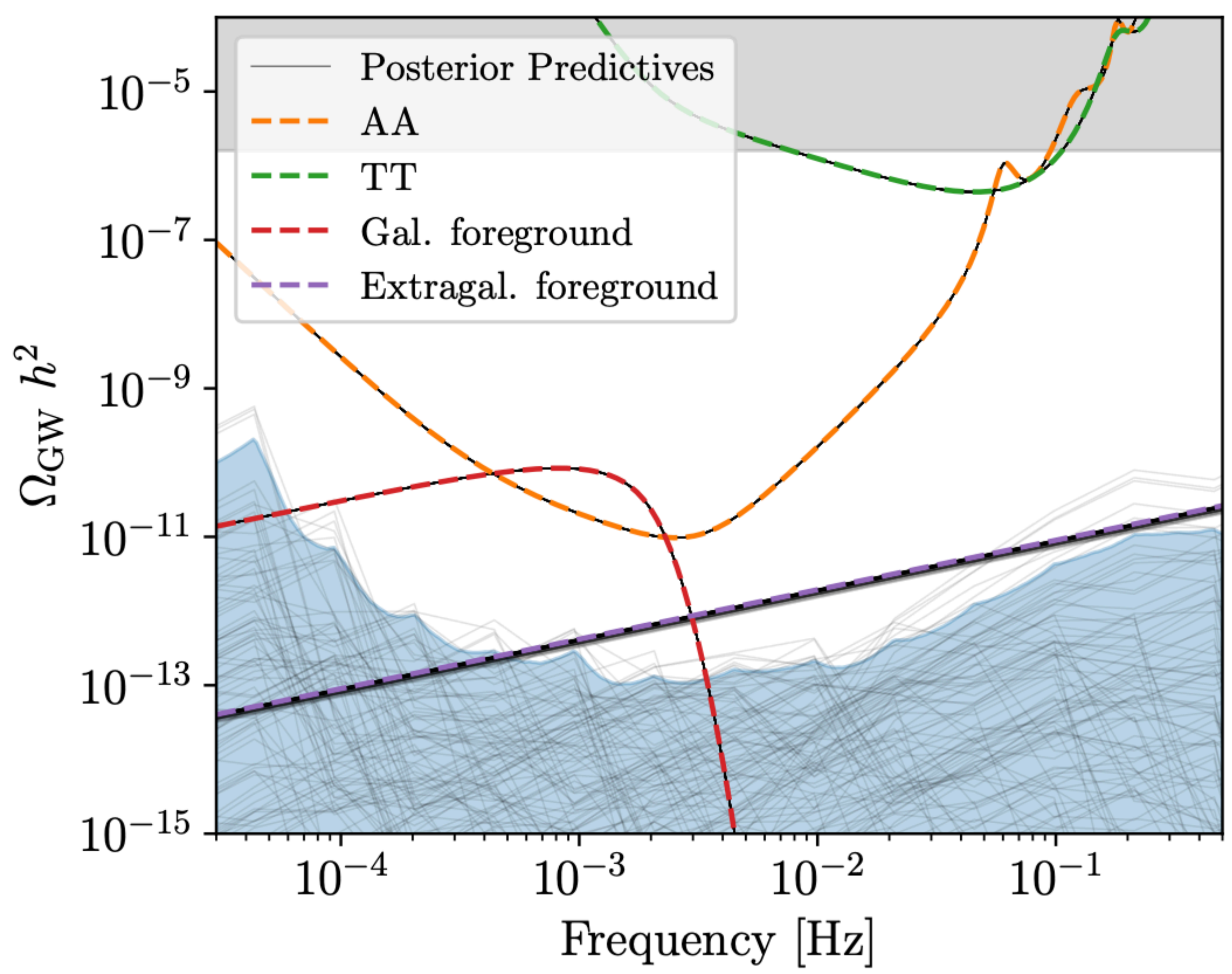
With the LISA CosWG, we have developed a code named SGWBinner (main developer Mario Pieroni, UAM) to study the detectability and reconstruction of SGWBs.

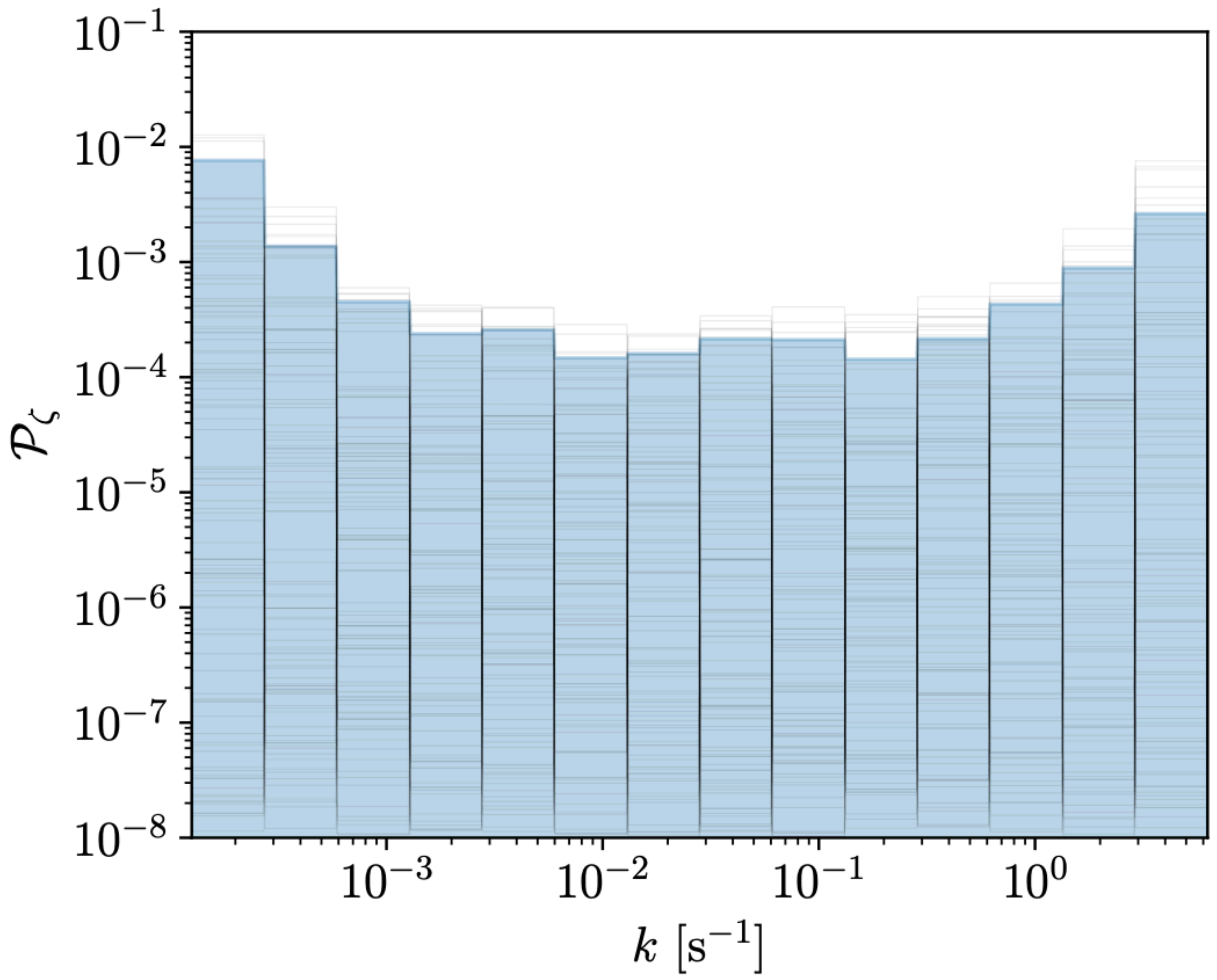
The analysis includes variations of galactic and extragalactic foregrounds. We assume stationary Gaussian noise and an equilateral LISA configuration; the results are robust to relaxing the equal-arm and equilateral assumptions.

Hartwig et al 2303.15929

Our pipeline also allows for the inclusion of scalar PNGs, as well as non-standard thermal histories at horizon re-entry.

SCALAR-INDUCED: LISA





We developed a pipeline to reconstruct the primordial scalar power spectrum from $\Omega_{GW}(f)$, enabling both template-based and free-form analyses.

LISA has the potential to place accurate constraints on the scalar power spectrum, either through a detection of the GWB or via upper limits.

Braglia et al 2407.04356 El Gammal et al 2501.11320

With the LISA CosWG, we have developed a code named SGWBinner (main developer Mario Pieroni, UAM) to study the detectability and reconstruction of SGWBs.

The analysis includes variations of galactic and extragalactic foregrounds. We assume stationary Gaussian noise and an equilateral LISA configuration; the results are robust to relaxing the equal-arm and equilateral assumptions.

Hartwig et al 2303.15929

Our pipeline also allows for the inclusion of scalar PNGs, as well as non-standard thermal histories at horizon re-entry.

Take-home messages

- Cosmological observations already place robust constraints on inflation, but progress may be limited within simple slow-roll scenarios.
- High-quality GW data are coming. While detections are not guaranteed, this offers a unique window on primordial physics—one we must be prepared to exploit.
- PTAs may already hint at primordial signals, motivating continued scrutiny and cross-checks with complementary experiments.
- LISA has tremendous potential: it is a low-risk mission for astrophysics, but a high-reward opportunity for probing primordial physics.